

Ordered alternative hypotheses

In some situations it is suspected that the groups follow an ordering under the alternative hypothesis, such as $\mu_1 \leq \mu_2 \leq \dots \leq \mu_k$ (which is equivalent to $F_1(x) \geq F_2(x) \geq \dots \geq F_k(x)$). A general idea is that we can get a more powerful test for detecting this alternative by taking tests for a one-sided alternative and using them together. In other words, if T_{ij} is a test statistic for detecting $H_a : F_i(x) \geq F_j(x)$, then $\sum_{i < j} T_{ij}$ can be used as a test statistic for detecting $H_a : F_1(x) \geq F_2(x) \geq \dots \geq F_k(x)$. The Jonckheere-Terpstra (JT) statistic is a sum of this type, where the T_{ij} are one-sided Mann-Whitney statistics. A permutation test can be used to obtain a p value for the JT test, or a large-sample normal approximation can be used with:

$$E(JT) = \frac{N^2 - \sum_{i=1}^k n_i^2}{4} \text{ and } Var(JT) = \frac{N^2(2N+3) - \sum_{i=1}^k n_i^2(2n_i+3)}{72}.$$

The JT test can be performed in SAS with Proc FREQ.