Effective Selection Coefficient of Meiotic Drive

Dick Gomulkiewicz Steve Krone

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Consider a diploid locus with alleles D and d. Let p be the frequency of D and $q = 1 - p$ the frequency of d among the parents. Assume the parental diploid genotype frequencies result from random mating, that is, the usual Hardy-Weinberg proportions.

Rate of spread by meiotic drive Suppose that D causes meiotic drive such that a fraction $\frac{1}{2} + \delta$ of the gametes produced by a heterozygote Dd parent are D and $\frac{1}{2} - \delta$ are d. The parameter δ measures the *segregation distortion* caused by D. We have $0 \leq \delta \leq \frac{1}{2}$, and $\delta = 0$ gives the usual Mendelian segregation ratio. (The quantity $e = 2\delta$ is sometimes called the drive *conversion efficiency*, and is equal to the probability that a d allele is converted to a D allele.) If we assume that there are no fitness differences conferred by the two alleles, the frequency of D among the gametes produced by all parents is

$$p^* = 1 \cdot p^2 + (\tfrac{1}{2} + \delta) \cdot 2pq = p + 2\delta pq \tag{1}$$

The next generation is formed by random mating, which is equivalent to random union of gametes but with allele frequencies p^* and $q^* = 1 - p^*$. The resulting offspring are in Hardy-Weinberg proportions with frequencies $p' = p^* = p + 2\delta pq$ and $q' = 1 - p'$ of D and d, respectively. The rate of evolution due to meiotic drive is thus

$$\Delta_D p = p' - p = (p + 2\delta pq) - p = pq \cdot 2\delta \tag{2}$$

If $\delta > 0$, the drive allele D will sweep to fixation.

Rate of spread by selection To get a sense of how fast an allele spreads by drive compared to one spread by natural selection alone, consider the rate of spread of D when it is favored and fitnesses are "additive" such that the respective fitnesses of DD, Dd, and dd are 1, $1 - s/2$, and $1 - s$, with selection coefficient $s > 0$. (In this calculation we are pretending that D is not a drive allele.) Equations (3.2) and (3.3) on p. 64 of Gillespie with $h = \frac{1}{2}$ show that D spreads at per generation rate

$$\Delta_s p = pq \cdot \frac{s}{2(1 - 2qs)} \tag{3}$$

Meiotic drive versus selection What selection coefficient s corresponds to meiotic drive with segregation distortion δ? In other words, what s makes $\Delta_s p = \Delta_D p$ when the current frequency of D is p? Equations (2) and (3) show

that both rates depend on the genetic variation pq; they will thus be the same if the 'differentials' match, i.e., if

$$2\delta = \frac{s}{2(1-2qs)} \tag{4}$$

The meiotic drive differential 2δ is constant regardless of allele frequency whereas the selection differential—i.e., the right-hand side of (4)—depends on q as well as s. Notice that the selection differential increases from $s/2$ when $q = 0$ to $s_{e(D)} := s/[2(1-s)]$ when $q = 1$. Selection* will thus be at least as effective as meiotic drive over all allele frequencies if the selection coefficient for D satisfies $s \geq s_{e(D)}$.

To find $s_{e(D)}$ for a given distortion δ, set

$$2\delta = \frac{s_{e(D)}}{2[1-2s_{e(D)}]} \tag{5}$$

and solve for $s_{e(D)}$. After some algebra, this gives

$$s_{e(D)} = \frac{4\delta}{1+8\delta} \tag{6}$$

Meiotic drive with complete distortion, $\delta = 0.5$, is comparable to natural selection with minimum selection coefficient $s_{e(D)} = 0.4$. Bearing in mind that selection coefficients are typically 0.01 up to 0.1, this is shows meiotic drive is capable of being anywhere from 4 to 40 times as powerful as natural selection.

Another way to appreciate the potential efficacy of meiotic drive is in terms of the time needed for a favored allele to sweep from a very low starting frequency to near fixation. For natural selection, this time is proportional to $1/s$ generations. For meiotic drive with complete distortion, the sweep time is proportional to $1/0.4 = 2.5$ generations!

*Think of $s_{e(D)}$ as the "drive effective" selection coefficient, that is, the *smallest* selection coefficient that would generate the same rate of evolution as meiotic drive with distortion δ.