

Analysis of a Beam in Pure Bending

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There are four basic equations that govern the theory of pure bending. These are derived from analyzing a beam as it is subject to a load P acting at one end. In our poster we cover the four main parameters that apply to this theory, as well as some additional information that can be applied to a beam subjected to pure bending.

The first relationship is called the strain curvature theory. As a beam is bent it curves around a theoretical O' , known as the center of curvature. The distance from each point on the beam to the center of curvature is called the radius of curvature and is designated as ρ , rho. For small deflections we can say that a small distance on the curve, ds , is equal to a small deflection, dx , along the beam and can be defined by $dx = \rho * d\theta$, where θ is the angle of the section. As a beam is bent it deforms in either tension or compression, except for the neutral axis which will remain unchanged. Using this we can determine what the strain will be at any position along the beam. Taking the radius of curvature of a point y_1 with respect to the neutral axis, we find that $\text{radius} = \rho - y_1$. We can find the new length of this section using $L = \text{radius} * d\theta$. By rearranging these equations we come up with $L - dx = (-y / \rho) * dx$, where dx is the original length and L is the new length. $L - dx$ gives us the change in length of the beam at position y , and if we divide dx out of the second part of the equation we get the change in the length of the beam divided by the original length, resulting in a strain. The final equation comes out to look like:
 $\epsilon = -y / \rho$.

The second equation in our analysis is called the moment-curvature equation. Since an element in pure bending is subject to no tensile stress, the only stress acting on a member is due to the moment, where the force acting at each point is equal to $\sigma * dA$. We know that $\sigma = E * \epsilon$ (Hook's law), and that $\epsilon = -y / \rho$, from above. By combining these

equations we get that the $\int (-E/\rho)*y*dA=0$, since the sum of the forces is zero. We also know that the internal bending moment created has to be equal to the external moment. From this we get that the external moment M is equal to the sum of the moments created by σ , thus $M = \int \sigma*y*dA$, which converts to $(-E/\rho)$ multiplied by the $\int y^2*dA$, which is the second moment of the area, or also known as I . By combining these equations we come up with the expression $M = (E/\rho)*I$, or $1/\rho = M/(E*I)$.

We can derive the strain equation by combining the two relationships that was just derived and Hook's law. We know that $\epsilon = -y/\rho$, $\sigma = E*\epsilon$, and $1/\rho = M/(E*I)$. First we solve for ρ , $\rho = -y/\epsilon$. Then we isolate ϵ , $\epsilon = \sigma/E$. By substituting ϵ into the equation for ρ and then ρ into the moment curvature equation E cancels out and we end up with $\sigma = -M*y/I$.

Finally we solve for the internal energy of a beam in pure bending. We know that the energy stored in a beam in bending is $dU = (M/2)*d\theta$. We already demonstrated that $d\theta = ds/\rho$, so we can substitute this into the internal energy equation and get $dU = (M/2)*ds/\rho$. Next we substituted the moment of curvature equation for ρ and come up with the expression $dU = M^2*ds/(2*E*I)$. Since we know that for small deflections $dx = ds$, we can substitute and integrate the equation to get the energy of a beam in bending, $U = M^2*dx/(2*E*I)$.

We also included the stress analysis for composite beams in our poster because it may be useful information, since composite beams can be used quite often in design.

We had three sources for our information. The primary text we used was the James M. Gere, sixth edition, Mechanics of Materials textbook, pages 300-312. We found the information for internal energy on pages 109 and 110 of the Shigley and

Mischke Mechanical Engineering Design text. Lastly we used an analysis of pure bending handout provided by Edwin Odem.