FAILURE CRITERIA HANDOUT

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Example 1)

An ASTM 30 cast iron has a minimum ultimate strength of 30 kpsi in tension and 100 kpsi in compression. Find the factor of safety if the cast iron is loaded as follows:

$\sigma_x := 20$ kpsi $\qquad \sigma_y := -40$

Note that we have a BRITTLE material where

$S_{ut} := 30$ kpsi $\qquad S_{uc} := 100$ kpsi $\qquad S_{ut} \neq S_{uc}$

Which will lead us to use the Mod. II Mohr theory.

$0 \le \sigma_1 \le S_{ut} \qquad -S_{uc} \le \sigma_3 \le S_{ut}$

Through Morh's circle analysis:

$\sigma_1 := 20$ kpsi $\qquad \sigma_3 := -40$ kpsi

$$\frac{n \cdot \sigma_1}{S_{ut}} = 1 - \left(\frac{n \cdot \sigma_3 + S_{ut}}{-S_{uc} + S_{ut}}\right)^2$$

From this expression $n := 1.333$

Example 2)

A T6 319 cast aluminum fixture is loaded as follows:

$\sigma_x := 72$ kpsi $\qquad \sigma_y := 12$ kpsi

Note that we have a BRITTLE material where

$S_{ut} := S_{uc} \qquad S_{ut} := 36$

Which will lead us to use the Maximum-Normal-Stress Hypothesis theory.

Through Morh's circle analysis:

$\sigma_1 := 72$ kpsi $\qquad \sigma_3 := 12$ kpsi

$|\sigma_1| \ge |\sigma_3|$ Therefore $n := \dfrac{S_{ut}}{\sigma_1}$

From this expression: $n := 0.5$

Note that an n<1 signifies failure of the component

Example 3)

This problem was taken from Shigley's Mechanical Engineering Design 5th Ed.

A hot-rolled bar of ductile material has a minimum yield strength in tension and compression of 44 kpsi. Find the factors of safety for each applicable theory of failure for the following stress states.

d) σ_x = 11 kpsi, σ_y = 4 kpsi, τxy = 1 kpsi cw

Note that we have a DUCTILE material where

$S_{ut} := S_{uc}$ $\quad S_{ut} := 36$

Which will lead us to use the Distortion-Energy Hypothesis

Through Morh's circle analysis:

$\sigma_1 := 11.14$ kpsi $\quad \sigma_2 := 3.859$ kpsi $\quad \sigma_3 := 0$

$$\sigma' := \sqrt{\frac{\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}{2}}$$

$\sigma' := 9.798$ kpsi

$$n := \frac{S_{ut}}{\sigma'}$$

From this expression $n := 3.67$

Example 4)

A ductile component with an ultimate compressive strength of 100 kpsi and an ultimate tensile strength of 50 kpsi is loaded as follows:

$\sigma_x := 20$ kpsi $\quad \sigma_y := -30$ kpsi $\quad \tau_{xy} := 15$

Note that we have a DUCTILE material where

$S_{ut} \neq S_{uc}$

Which will lead us to use the Coulomb Mohr Hypothesis.

Through Morh's circle analysis:

$\sigma_1 := 24.15$ kpsi $\quad \sigma_2 := 0$ kpsi

Because $\sigma_1 > 0$ and $\sigma_3 < 0$ the material is in quadrent 3

Therefore n can be found from the equation

$$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} := \frac{1}{n}$$

From this expression $n := 1.21$