

## Dynamic Modulus ( $E^*$ ) for Mix and Pavement Design

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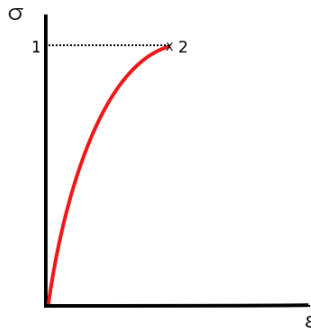
## Outline

- What is  $E^*$ ?, Why  $E^*$  not just  $E$ ?
- How  $E^*$  is measured?
- Model Development:
  - Mix parameters
  - Mathematical formulation
  - Regression analysis
  - Validation using field mixes
  - Analysis using MEPDG
- Application
  - Software



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## The Elastic Modulus, $E$

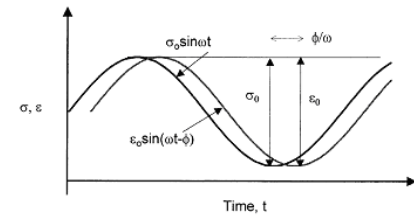


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## The Dynamic Modulus ( $E^*$ )

The dynamic modulus is defined as the absolute value of the Complex Modulus ( $E^*$ ), which is the ratio of stress / strain for a linear **visco-elastic** material.

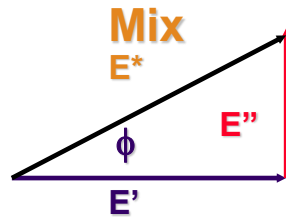
$$|E^*| = \frac{\sigma_o}{\epsilon_o}$$



### Why $E^*$ ?

$E^*$  (Dynamic modulus) accounts for the viscous component as well.

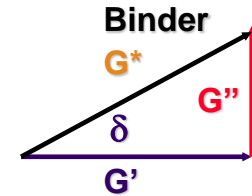
$E$  (elastic modulus)  $\cong E'$  when the phase angle  $\phi$  is zero.



### Why $E^*$ ?

$E^*$  is a function of:  
Binder  $G^*$ , and  
Aggregate properties

Most likely the viscous element will be affected by the binder, while the elastic component by both (binder and aggregates)



### The Dynamic Modulus ( $E^*$ ) Test

- AASHTO TP 62-03 Test Protocol.
- Test Temperatures: 14, 40, 70, 100 and 130 °F.
- Loading frequencies: 0.1, 0.5, 1, 5, 10 and 25 Hz @ each temperature.



### $E^*$ Sample Preparation



## Dynamic Modulus Test



## Typical Results

$E^*$  (MPa) vs. Temp and Frequency

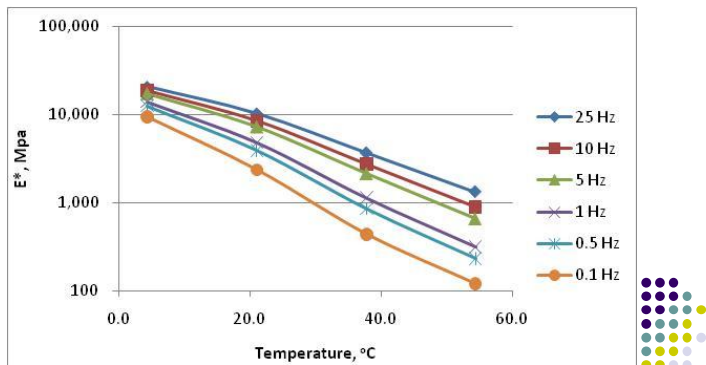
Temp, °C	25 Hz	10 Hz	5 Hz	1 Hz	0.5 Hz	0.1 Hz
4.4	20,683	18,784	17,345	14,070	12,619	9,531
21.1	10,257	8,518	7,284	4,846	3,964	2,383
37.8	3,694	2,743	2,145	1,137	863	442
54.4	1,331	895	660	318	235	121



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## Typical Results

$E^*$  vs. Temp and Frequency



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## $E^*$ Modeling



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## Empirical Models

- Asphalt Institute Method (Shook and Kallas 1969)
- Witczak Models (Witczak 1978, Miller et al. 1983, Witczak and Fonesca 1996 & Bari and Witczak 2006)
- Christensen et al. (2003)



## Witczak's 1996 Model

$$\log E^* = -1.25 + 0.029P_{200} - 0.0018(P_{200})^2 \dots$$

$$+ 0.0028P_4 - 0.058V_a - 0.822 \frac{V_{beff}}{V_{beff} + V_a} \dots$$

$$+ \frac{3.872 + 0.0021P_4 + 0.004P_{38} - 0.000017(P_{38})^2 + 0.0055P_{34}}{1 + \exp(-0.603313 - 0.313351 \log f - 0.393532 \log \eta)}$$

where,

$E$  = dynamic modulus of mix,  $10^5$  psi.

$P_{200}$  = percent aggregate passing #200 sieve.

$P_4$  = percent aggregate retained on #4 sieve.

$P_{38}$  = percent aggregate retained on 3/8 inch sieve.

$P_{34}$  = percent aggregate retained on 3/4 inch sieve.

$V_a$  = percent of air voids in the mix by volume.

$V_{beff}$  = percent of effective binder content by volume.

$f$  = loading frequency, Hz.

$\eta$  = asphalt viscosity at any temperature and degree of aging,  $10^6$  poises.

**This Model is  
used in ME-PDG  
Level 3**



## Witczak's 2006 Model

$$\log_{10} E^* = -0.349 + 0.754(|G_b^*|^{-0.0052}) \dots$$

$$\times \left( 6.65 + 0.032P_{200} + 0.0027(P_{200})^2 + 0.011P_4 - 0.0001(P_4)^2 \right) \dots$$

$$+ 0.006P_{38} - 0.00014(P_{38})^2 - 0.08V_a - 1.06 \frac{V_{beff}}{V_{beff} + V_a} \dots$$

$$+ \frac{2.56 + 0.03V_a + 0.71 \frac{V_{beff}}{V_{beff} + V_a} + 0.012P_{38} - 0.0001(P_{38})^2 + 0.01P_{34}}{1 + \exp(-0.7814 - 0.5785 \log |G_b^*| + 0.8834 \log \delta_b)}$$

where,

$G_b^*$  = binder's dynamic shear modulus

$\delta_b$  = binder's phase angle



## Our Goal

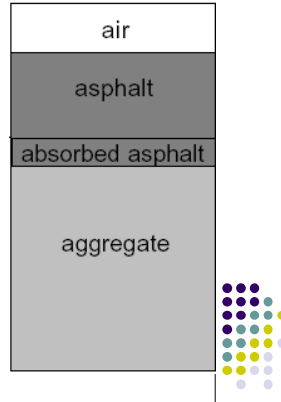
was to develop a **simple model** that  
predicts HMA Dynamic Modulus ( $E^*$ )  
using the constituents of the asphalt mix  
and their interactions.



## Factors Affecting the Dynamic Modulus of Asphalt Mixes

- Binder
- Aggregates
- Air voids

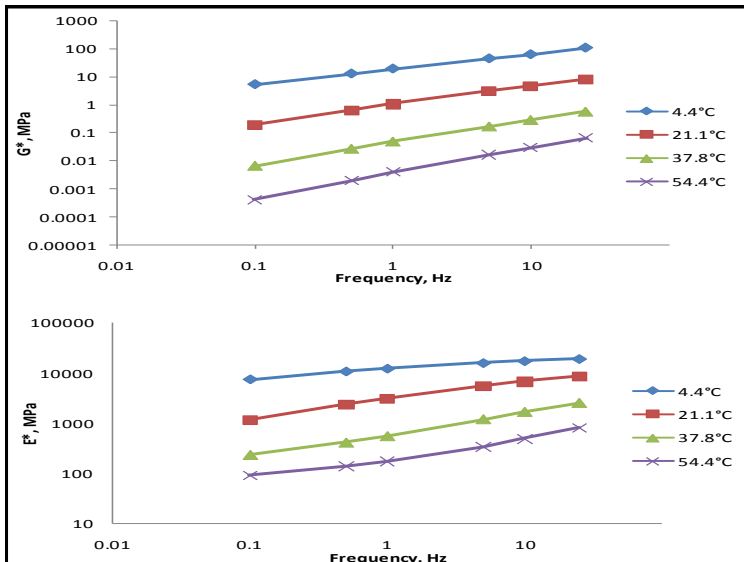
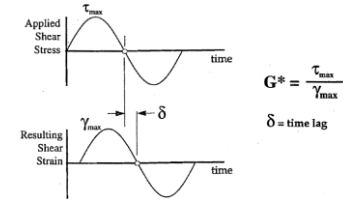
(Interaction?)



## Asphalt Binder Properties

Asphalt Binder Shear Modulus ( $G^*$ ).

Measured by the Dynamic Shear Rheometer.

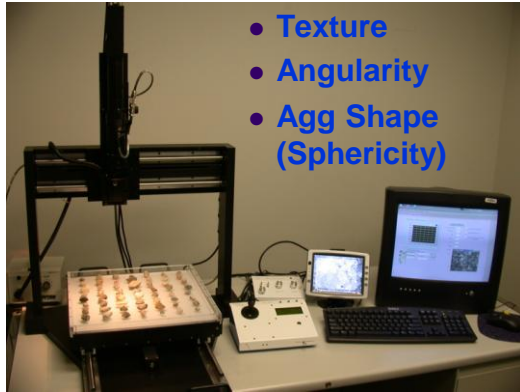


## Aggregates Properties

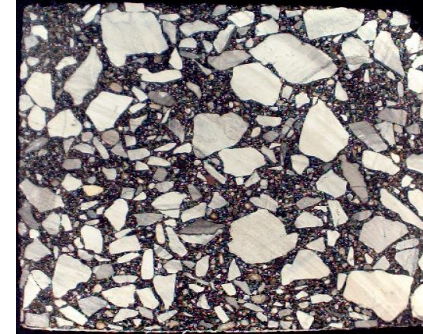
- Size,
- Shape (e.g. Angularity),
- Texture, and
- Structure.



## Aggregate Shape Characteristics and CEI



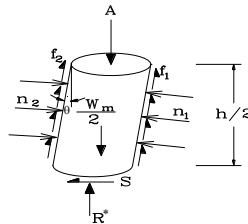
## Mix Image



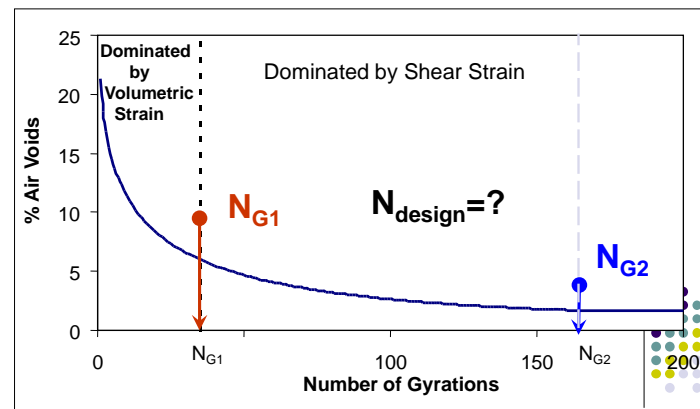
## Aggregates Properties

Measured by the  
Gyratory Stability (GS)

$$\text{Gyratory Stability, GS} = \frac{N_{\text{design}}}{N_{G1}} \cdot S_{N\theta} \cdot d_e$$



## Analysis of the Compaction Curve

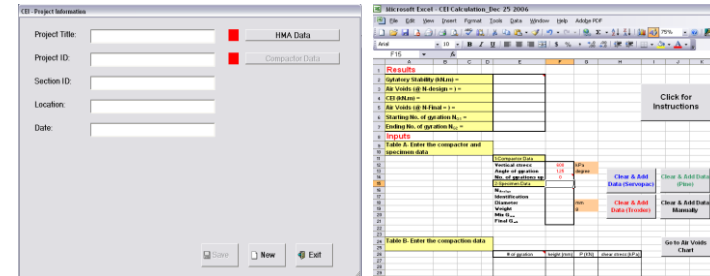


## Gyratory Stability

$$GS = \sum_{N_{G1}}^{N_{design}} S_{N\theta} \cdot \Delta d_e$$



## Gyratory Stability Software



Visual Basic Software

Excel File w/ Macros

## Model Formulation

Variables	Units	Dimensions (L,M,T)
Asphalt Mix Dynamic Modulus, $E^*$	MPa	$ML^{-1}T^{-2}$
Binder Dynamic Shear Modulus, $G^*$	MPa	$ML^{-1}T^{-2}$
Gyratory Stability, GS (Compaction Energy)	kN.m	$ML^2T^{-2}$
Air voids, AV%	-	-
Binder Content, $P_b$ Or Aggregates Content, $P_s$	-	-

**n = 5**

**m = 3**

## PI Groups

Dimensional product =  $n - m = 5 - 3 = 2$  ( $\pi_1$  and  $\pi_2$ ).

- Binder effects:

$$\pi_1 = \frac{P_b \cdot E^*}{G^*} = \frac{ML^{-1}T^{-2}}{ML^{-1}T^{-2}}$$

- Aggregates effects:

$$\pi_2 = \frac{P_s \cdot E^*}{GS} = \frac{ML^{-1}T^{-2}}{ML^2T^{-2}} = L^{-3}$$

### PI Groups (cont.)

$\pi_2$  must be dimensionless. Therefore,

$$\pi_2 = \frac{P_s \cdot E^*}{GS} \cdot V = \frac{ML^{-1}T^{-2}}{ML^2T^{-2}} \cdot L^3$$

$$V = V_{se} + V_b + V_v$$

$$V = \frac{P_s}{G_{se} \cdot \rho_w} + \frac{P_b}{G_b \cdot \rho_w} + AV\% \cdot V$$

$$G_{mm} = \frac{1}{\frac{P_s}{G_{se}} + \frac{P_b}{G_b}}$$

$$V = \frac{1}{\rho_w G_{mm} \cdot (1 - AV\%)} = \frac{1}{\rho_w G_{mb}}$$

### PI Groups (cont.)

In summary,

$$\pi_1 = \frac{P_b \cdot E^*}{G^*}$$

$$\pi_2 = \frac{(1 - P_b) \cdot E^*}{\rho_w \cdot GS \cdot G_{mb}}$$

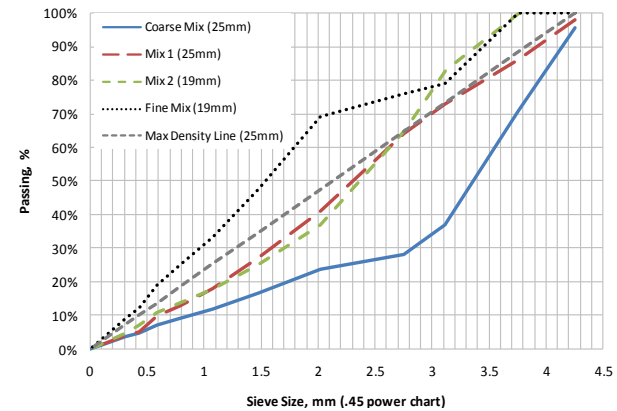
$$\pi_1 = f(\pi_2) \rightarrow \frac{P_b \cdot E^*}{G^*} = f\left(\frac{(1 - P_b) \cdot E^*}{\rho_w \cdot GS \cdot G_{mb}}\right) = \frac{1}{\frac{(1 - P_b) \cdot E^*}{\rho_w \cdot GS \cdot G_{mb}}}$$

$$E^* = a \left( \frac{G^*}{P_b} \cdot \frac{\rho_w \cdot GS \cdot G_{mb}}{(1 - P_b)} \right)^b$$

### Lab Mix Matrix

Asphalt Content	-0.5	Opt	0.5	-0.5	Opt	0.5	-0.5	Opt	0.5
Binder Grade	PG 70-34			PG 70-28			PG 70-22		
Coarse Mix					√				
Mix 1		√		√	√	√			√
Mix 2		√			√				
Fine Mix					√				
	PG 64-34			PG 64-28			PG 64-22		
Mix 1					√				
Mix 2	√	√	√		√				√
	PG 58-34			PG 58-28					
Mix 1					√				
Mix 2		√							

### Lab Mixes – Selected Aggregate Structures





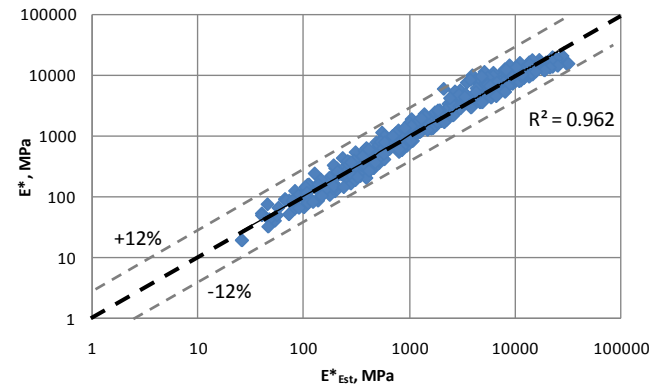
## Proposed Model

$$E^* = 1.08 \left( \frac{G^* \cdot GS \cdot G_{mb}}{P_b (1 - P_b)} \right)^{0.5583}$$

where,

E\*: Dynamic Modulus for Asphalt Mix, MPa,  
 G\*: Dynamic Shear Modulus for RTFO Aged Binder, MPa,  
 P<sub>b</sub>: Percent Binder Content,  
 GS: Gyrotory Stability, kN.m,  
 G<sub>mb</sub>: Bulk Specific gravity of Mix, G<sub>mb</sub> = G<sub>mm</sub> (1-AV%),  
 G<sub>mm</sub>: Maximum Specific gravity of Mix, and  
 AV%: Air Voids.

## Data Analysis

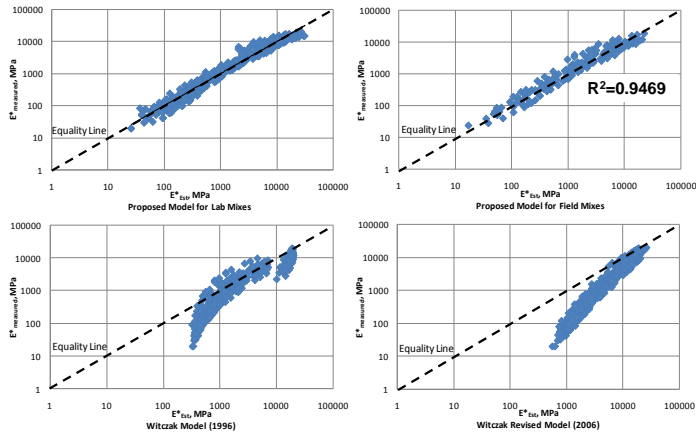


## MODEL VALIDATION

## Field Mixes

Mix ID	1	2	3	4	5	6	7
Class	SP4	SP3	SP3	SP2	SP3	SP3	SP3
ESALs	> 30x10 <sup>6</sup>	3 - 30x10 <sup>6</sup>	3 - 30x10 <sup>6</sup>	0.3 - 3x10 <sup>6</sup>	3 - 30x10 <sup>6</sup>	3 - 30x10 <sup>6</sup>	3 - 30x10 <sup>6</sup>
N-design	125	100	100	75	100	100	100
<b>Mix Properties</b>							
PQ	70-28	64-34	70-28	58-34	70-28	70-28	70-28
Gmm	2.449	2.424	2.568	2.480	2.448	2.581	2.460
P <sub>b</sub> %	4%	4%	4%	4%	4%	3.5%	3.5%
G <sub>b</sub>	1.021	1.025	1.034	1.009	1.021	1.036	1.036
G <sub>sb</sub>	2.586	2.558	2.771	2.731	2.589	2.822	2.822
<b>%Passing</b>							
25mm (1")	98%	100%	100%	100%	100%	100%	98%
19 mm (3/4")	86%	100%	97%	100%	100%	100%	90%
12.5mm (1/2")	73%	83%	83%	95%	95%	96%	74%
9.5mm (3/8")	64%	65%	71%	78%	86%	87%	66%
4.75mm (#4)	41%	37%	51%	53%	45%	58%	40%
2.36mm (#8)	27%	25%	34%	35%	32%	36%	25%
1.18mm (#16)	18%	18%	23%	22%	23%	22%	16%
600µm (#30)	13%	14%	16%	15%	16%	17%	12%
300µm (#60)	10%	11%	11%	12%	9%	13%	10%
150µm (#100)	5%	7%	8%	9%	5%	8%	7%
75µm (#200)	4.0%	4.7%	5.9%	6.8%	4.0%	6.4%	5.7%

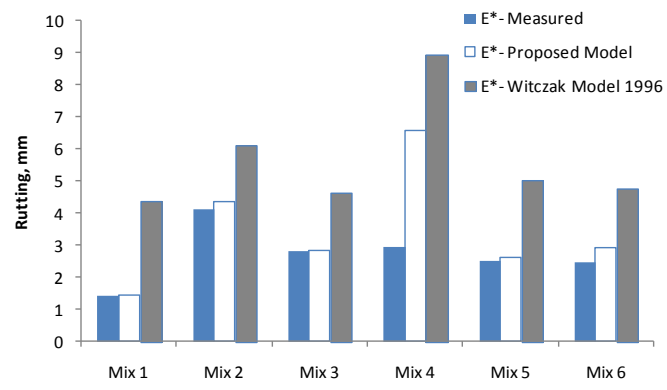
### $E^*$ Predicted vs. Measured



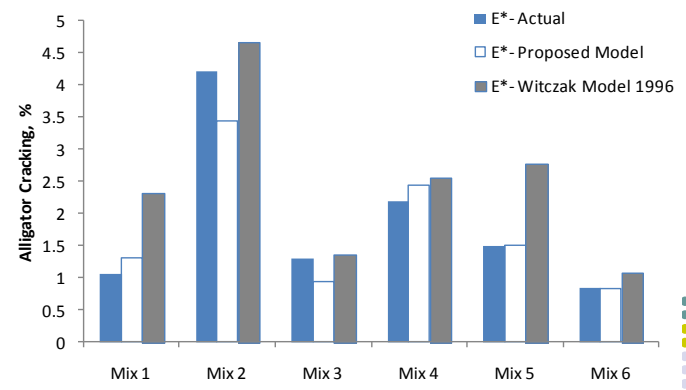
### Application to M-E PDG



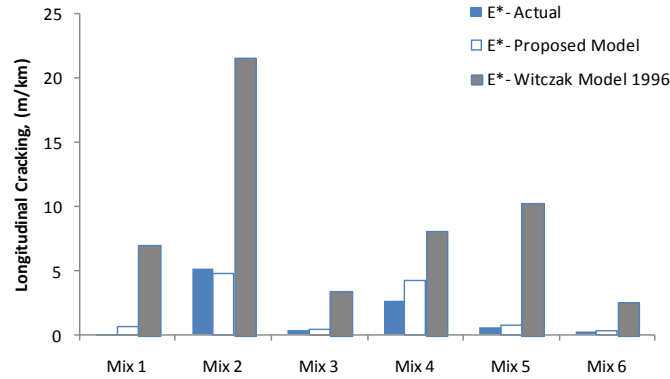
### Prediction of Permanent Deformation Using MEPDG



### Prediction of Alligator Cracking Using MEPDG



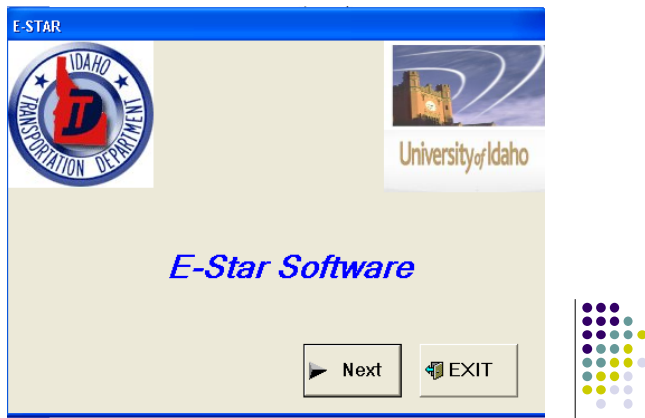
## Prediction of Longitudinal Cracking Using MEPDG



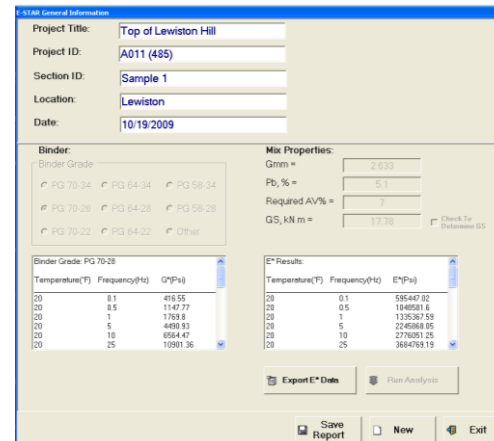
## In Summary

1.  $E^*$  is a mix property that depends on **binder grade and content**, **aggregates properties** and **mix structure**.
2. Dimensional analysis was used to develop the mathematical form of the  $E^*$  model. The model was found to be dependent on: **GS**, **Pb**, **Gmb**, and **Binder  $G^*$** .
3. Regression analysis showed  $R^2$  equal to 0.962.
4. MEPDG runs showed predicted distresses with  $E^*$  from the model were closer to the measured values than Level 3.

## $E^*$ Software



## $E^*$ Software Output



## Acknowledgement

- External funds for this project was provided by the Idaho Transportation Department (ITD) and the US Department of Transportation. Internal in-kind funds and partial support for testing equipment was provided by the University of Idaho.
- We acknowledge support provided by all these organizations:
  - US DOT
  - ITD
  - UI – NIATT
  - UI – Civil Engineering



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## Project Team

- **Fouad Bayomy (PI)**,  
Phase A – Deformation Studies, and overall project phases
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Phase B: Fracture and Fatigue studies
- **Richard Nielsen (Co-PI)**,  
Reliability Studies
- **Thomas Weaver (Co-PI)**,  
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- **Graduate Students**
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  - Baek, Seung Il
  - Prashant Darveshi
- **ITD Coordinator (s)**
  - Mike Santi (Main Contact)
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- **USDOT**
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  - Ed Weiner, COTR
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  - Paul Ziman (FHWA, Boise Office)
- **External Testing**
  - Idaho Asphalt Supply, Inc.
- **NIATT and CE Support**
  - Judy LaLonde
  - Tami Noble
  - Don Parks
  - Undergraduate Students: Frank Eckwright



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## Questions?

