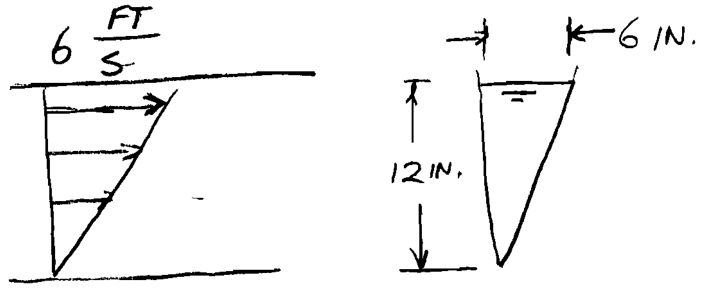
5.19

GIVEN:

VELOCITY PROFILE IN

V SHAPED CHANNEL

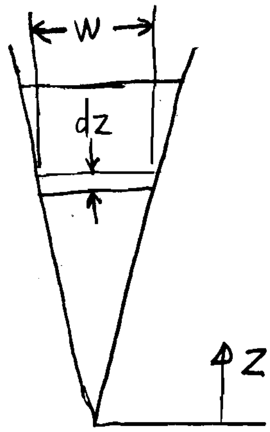


FIND: FLOW RATE

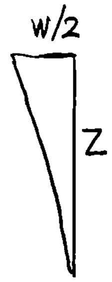
SOLUTION:

$$Q = \int_A V \, dA$$

$$dA = w \, dz$$



FIND W IN TERMS OF Z.



$$\frac{3}{12} = \frac{W/2}{Z}$$

$$6W = 3Z$$

$$w = \frac{1}{2} Z \qquad dA = \frac{1}{2} z \, dz$$

$$V(z) = \frac{Z}{12 \text{IN.}} (6 \text{ FT/s}) = \frac{Z}{1 \text{ FT}} (6 \text{ FT/s}) = 6 Z \ s^{-1}$$

$$Q = \int_0^{1 \text{FT}} 6 Z \left(\frac{1}{2} z \, dz\right) S^{-1} = 3 S^{-1} \int_0^{1 \text{FT}} z^2 dz$$

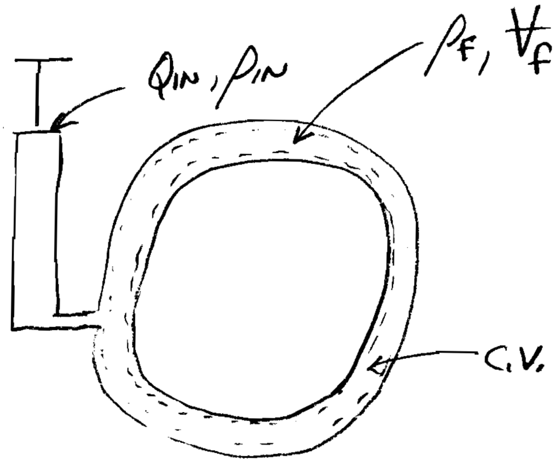
$$Q = 3 S^{-1} \cdot \frac{Z^3}{3} \Big|_0^{1 \text{FT}} = 1 \text{ FT}^3/S = 1 \text{ CFS}$$

5.32

GIVEN: A PUMP USED TO FILL A BIKE TIRE. DISCHARGE THROUGH PUMP IS 1 CFM. DENSITY OF AIR ENTERING PUMP IS 0.075 LBm/FT^3. INFLATED VOLUME OF TIRE IS 0.04 FT^3. DENSITY OF AIR IN INFLATED TIRE IS 0.4 LBm/FT^3.

FIND: TIME TO INFLATE TIRE, NO AIR IN TIRE INITIALLY.

SOLUTION: DRAW DONUT SHAPED C.S.
APPLY CONS. OF MASS.



TIME RATE OF CHANGE OF MASS IN C.V. + NET FLOW RATE OF MASS THROUGH C.S. = 0

$$\frac{d}{dt}\int_{CV}\rho\, d\forall + \int_{CS}\rho \underline{V}\cdot \underline{dA} = 0$$

ρ DOES NOT VARY SPATIALLY (WITH x, y, z) IN C.V. REMOVE ρ FROM INTEGRAL.

$$\frac{d}{dt}\rho\int_{CV} d\forall \qquad + \dot{m}_{OUT} - \dot{m}_{IN}$$

$$\frac{d}{dt}(\rho\forall) \qquad m(t) = \rho\forall(t)$$ ~ TOTAL MASS OF AIR IN TIRE AT TIME t.

$$\frac{dm}{dt} + \cancel{\dot{m}_{OUT}}^{\,0} - \dot{m}_{IN} = 0$$

$$\dot{m}_{IN} = \rho_{IN} Q_{IN} = 0.075\frac{LBm}{FT^3}\left(1\frac{FT^3}{MIN}\right) = 0.075\frac{LBm}{MIN}$$

$\dot{m}_{IN} = 0.075 \dfrac{LBm}{MIN}$ IS CONSTANT.

5.32 CONTINUED

$$\frac{dm}{dt} = \dot{m}_{IN}$$

$$\int dm = \int \dot{m}_{IN}\, dt$$

CONSTANT OF INTEGRATION

$$m = \dot{m}_{IN}\, t + C_1$$

AT $t=0$, $m = C_1$, SO C_1 IS THE MASS OF AIR IN THE TIRE AT $t=0$.

ASSUME THAT THIS IS ZERO.

$$m = \dot{m}_{IN}\, t$$

WHEN TIRE IS FULL $m = \forall_f \rho_f$

$$m = 0.04\,FT^3 \left(0.4\ \frac{LBm}{FT^3}\right)$$

$$m = 0.016\ LBm$$

$$t = \frac{0.016\ LBm}{0.075\ LBm/MIN} = 0.21\ MIN$$

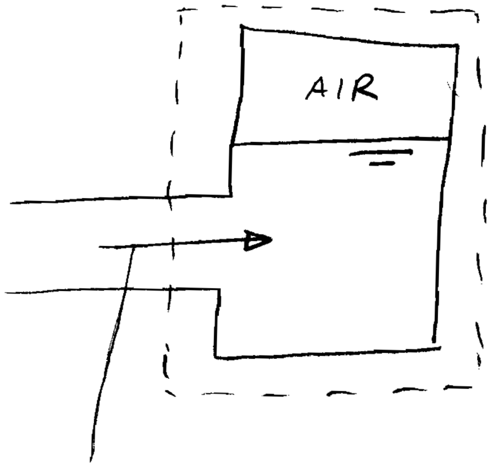
$$\boxed{t = 13\ \text{SECONDS}}$$

5.33

CONTINUITY → CONSERVATION OF MASS

GIVEN:

(a)

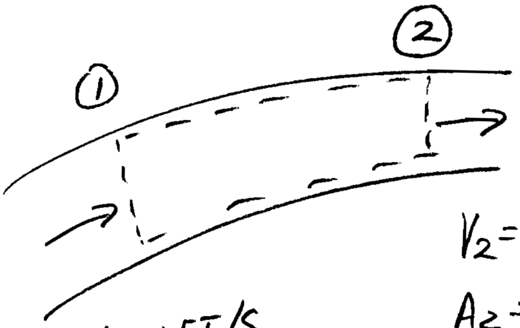


$V = 12$ FT/S

$A = 1.5$ FT^2

$\rho = 2$ SLUGS/FT^3

(b)



$V_1 = 1$ FT/S

$A_1 = 2$ FT^2

$\rho_1 = 2$ SLUGS/FT^3

$V_2 = 2$ FT/S

$A_2 = 1$ FT^2

$\rho_2 = 2$ SLUGS/FT^3

FIND: a. WHAT IS THE VALUE OF b?

$$\boxed{b = \frac{\text{MASS}}{\text{MASS}} = 1}$$ (a) AND (b)

b. DETERMINE $\frac{dB_{sys}}{dt}$.

$$\boxed{\frac{dB_{sys}}{dt} = \frac{dM_{sys}}{dt} = 0}$$ (a) AND (b)

THE MASS OF A MARKED OUT PORTION OF FLUID (A SYSTEM) DOES NOT CHANGE WITH TIME

5.33 CONTINUED

c. DETERMINE $\Sigma\, b \rho \underline{V} \cdot \underline{A}$.

$b = 1$ FOR BOTH (a) & (b).

(a) $\Sigma \rho \underline{V} \cdot \underline{A} = \rho V \hat{i} \cdot A \hat{n}$

$\hat{n} = -\hat{i}$ (UNIT OUTWARD NORMAL)

$$\Sigma \rho \underline{V} \cdot \underline{A} = -\rho V A$$

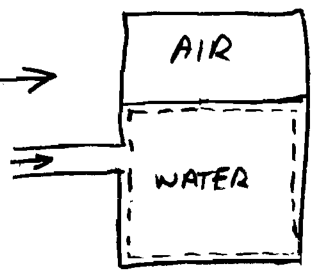
$$\boxed{\Sigma \rho \underline{V} \cdot \underline{A} = -36 \ \frac{\text{SLUGS}}{\text{S}}}$$

(b) $\Sigma \rho \underline{V} \cdot \underline{A} = -\rho_1 V_1 A_1 + \rho_2 V_2 A_2$

$$= -4 \ \frac{\text{SLUGS}}{\text{S}} + 4 \ \frac{\text{SLUGS}}{\text{S}}$$

$$\boxed{\Sigma \rho \underline{V} \cdot \underline{A} = 0}$$

d. REDRAW CONTROL SURFACE FOR (a): →



$$\frac{d}{dt} \int_{CV} b \rho \, d\forall = \frac{d}{dt} \rho \int_{CV} d\forall = \frac{d}{dt}(\rho \forall) = \frac{dm}{dt}$$

(a) $\frac{dm}{dt} - \dot{m}_{IN} + \dot{m}_{OUT} = 0$

$$\boxed{\frac{dm}{dt} = \dot{m}_{IN} = 36 \ \frac{\text{SLUGS}}{\text{S}}}$$

(b) $\frac{dm}{dt} - \dot{m}_{IN} + \dot{m}_{OUT} = 0$ $\quad \boxed{\frac{dm}{dt} = 0 \ \frac{\text{SLUGS}}{\text{S}}}$