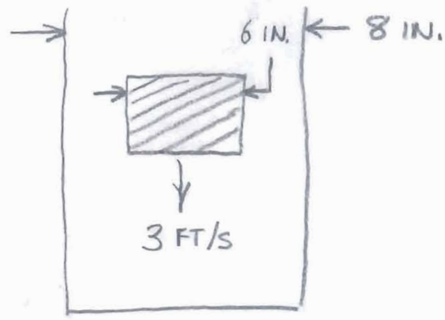
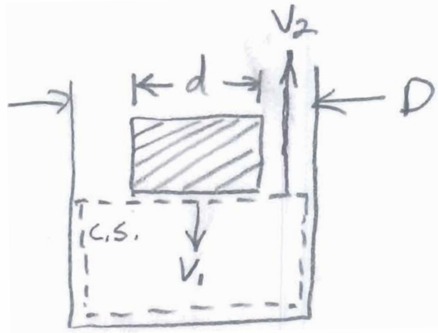
5.39 GIVEN: CYLINDER FALLING IN TUBE.

FIND: MEAN VELOCITY OF THE LIQUID IN THE SPACE BETWEEN THE CYLINDER AND THE WALL



SOLUTION: APPLY CONSERVATION OF MASS.

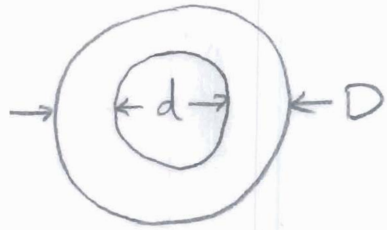
METHOD 1: DRAW C.S. AS SHOWN



$$\underbrace{\cancel{\frac{d}{dt}\int_{CV}\rho\, d\forall}}_{0} + \underbrace{\int_{CS}\rho \underline{V}\cdot d\underline{A}}_{} = 0$$

$$\cancel{\rho}\, Q_{OUT} - \cancel{\rho}\, Q_{IN} = 0$$

$$\rho = \text{CONSTANT} \Rightarrow Q_{OUT} = Q_{IN}$$



$$V_{IN} A_{IN} = V_{OUT} A_{OUT}$$

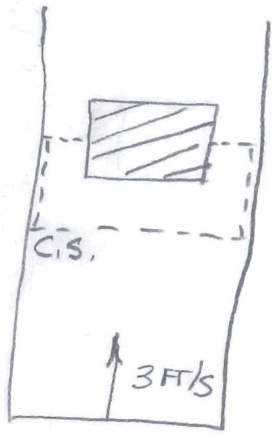
$$V_{OUT} = \frac{A_{IN}}{A_{OUT}} V_{IN} = \frac{\cancel{\frac{\pi}{4}}\, d^2}{\cancel{\frac{\pi}{4}}(D^2-d^2)} V_{IN}$$

$$V_{OUT} = \left(\frac{6^2}{8^2-6^2}\right) 3 \text{ FT/s} = 3.86 \text{ FT/s}$$

METHOD 2: DO A COORDINATE TRANSFORMATION SO THAT CYLINDER IS FIXED, BUT TUBE IS MOVING UPWARD A 3 FT/S.

DRAW C.S. AS SHOWN.

5.39 CONTINUED



$$V_{OUT} = \frac{A_{IN}}{A_{OUT}} V_{IN}$$

$$V_{OUT} = \frac{\cancel{\frac{\pi}{4}} D^2}{\cancel{\frac{\pi}{4}}(D^2-d^2)} V_{IN}$$

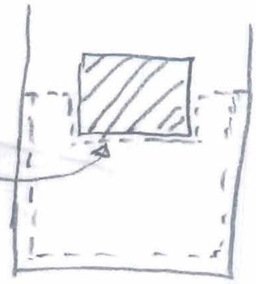
$$V_{OUT} = \left(\frac{8^2}{8^2-6^2}\right) 3\ FT/S$$

$V_{OUT} = 6.86\ FT/S$ IN MOVING TUBE COORDINATES

NOW TRANSFORM BACK TO FIXED TUBE COORDINATES

$$V_{OUT} = 6.86 - 3 = 3.86\ FT/S$$

METHOD 3: CONSIDER A CONTROL VOLUME THAT CHANGES WITH TIME.
THIS PART OF THE C.S. MOVES DOWN WITH TIME:



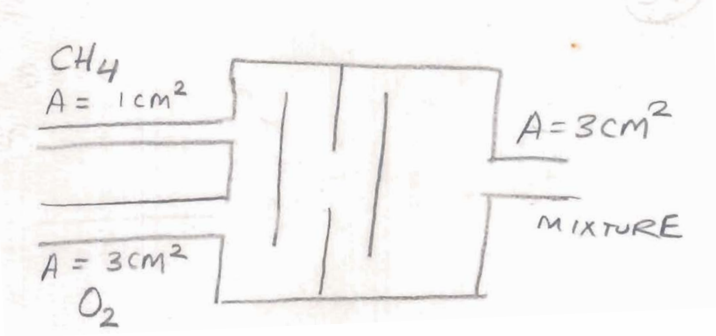
$$\frac{d}{dt}\int_{CV} \rho\, d\forall + \int_{CS} \rho \underline{V} \cdot \underline{dA} = 0$$

$$\frac{d\forall}{dt} + Q_{OUT} = 0$$

$$V_{OUT} A_{OUT} = -\frac{d\forall}{dt}$$

$$V_{OUT} = -\frac{d\forall}{dt}\frac{1}{A_{OUT}}$$

~~4.88~~ 5.55



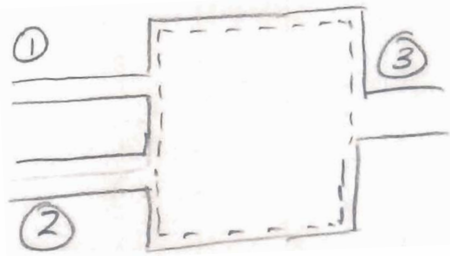
GIVEN: GAS MIXER WITH O_2 AND CH_4 AT 200 kPa AND 100°C.

$V_{O_2} = V_{CH_4} = 5\ m/s$. $\rho_{MIX} = 2.2\ kg/m^3$

FIND: VELOCITY OF GAS MIXTURE

SOLUTION:

$$\cancelto{0}{\frac{d}{dt}\int_{C.V.} \rho\, d\forall} + \cancelto{}{\int_{C.S.} \rho\, \underline{V} \cdot \underline{dA}} = 0$$



$$-\dot{m}_{IN} + \dot{m}_{OUT} = 0$$

$$\rho_1 = \frac{p_1}{R T_1} = \frac{200{,}000}{(518)(373)}$$

$$\rho_1 = 1.035\ \frac{kg}{m^3}$$

$$-\dot{m}_1 - \dot{m}_2 + \dot{m}_3 = 0$$

$$\rho_2 = \frac{200{,}000}{(260)(373)}$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

$$\rho_3 A_3 V_3 = \rho_1 A_1 V_1 + \rho_2 A_2 V_2$$

$$\rho_2 = 2.062\ \frac{kg}{m^3}$$

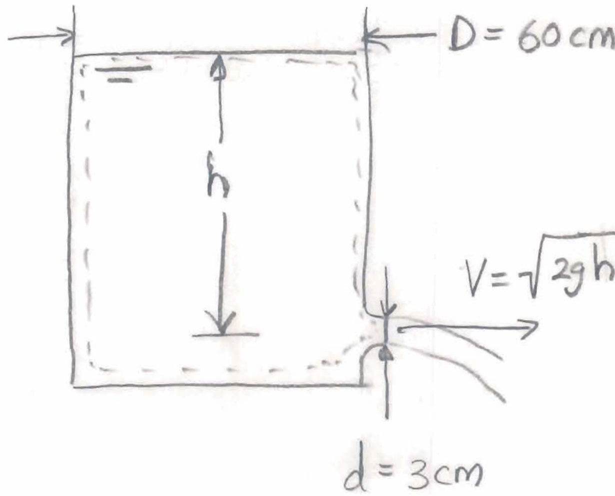
$$V_3 = \frac{\rho_1 A_1 V_1 + \rho_2 A_2 V_2}{\rho_3 A_3} = \frac{(1.035)(.0001)(5) + (2.062)(.0003)(5)}{(2.2)(.0003)}$$

$$\boxed{V_3 = 5.47\ m/s}$$

~~4.33~~ 5.59

GIVEN: WATER TANK AS SHOWN.

FIND: TIME FOR SURFACE TO DROP FROM $h = 3m$ TO $h = 50cm$



SOLUTION: $\frac{d(\rho \forall)}{dt} = -\dot{m}_{OUT}$

$\frac{d\forall}{dt} = -Q_{OUT} = -\frac{\pi}{4} d^2 \sqrt{2gh}$ h IS NOT CONST!

$\forall = h \frac{\pi}{4} D^2$ $\frac{d\forall}{dt} = \frac{\pi}{4} D^2 \frac{dh}{dt}$

$\cancel{\frac{\pi}{4}} D^2 \frac{dh}{dt} = -\cancel{\frac{\pi}{4}} d^2 \sqrt{2gh}$

$\int_{h_i}^{h_f} \frac{dh}{\sqrt{h}} = \int_0^t -\frac{d^2}{D^2} \sqrt{2g}\, dt$

$2(\sqrt{h_f} - \sqrt{h_i}) = -\frac{d^2}{D^2} \sqrt{2g}\, t$

$t = 2\left(\frac{D^2}{d^2}\right) \frac{\sqrt{h_i} - \sqrt{h_f}}{\sqrt{2g}} = 2\left(\frac{60}{3}\right)^2 \frac{\sqrt{300} - \sqrt{50}}{\sqrt{2(980)}}$

$\boxed{t = 185 s = 3 \text{ min } 54 s}$