ENGR335 **THE FOUNTAIN EXPERIMENT** (HW set 13) FALL 2006

1) Make a sketch of the fountain apparatus. Label all pertinent dimensions.

2) We will measure the height attained by the fountain in class for two different tank levels. Record these values.

3) Calculate the exit velocity at the nozzle by the Bernoulli equation.

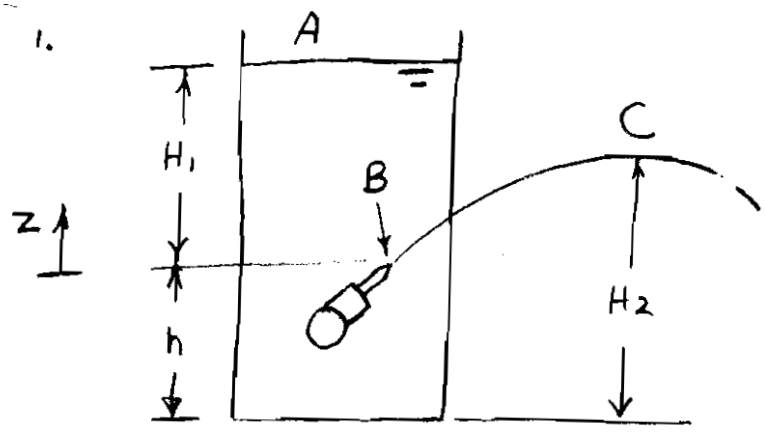
4) Calculate the theoretical height of the water stream by applying the Bernoulli equation. Do you expect the theoretical height to be greater than, equal to, or less that the measured height? Why? Compare your theoretical results to the measured results and discuss.

5) The water stream has a certain diameter at the nozzle, D_{noz}. It also has a diameter at the point where it passes through the maximum height, D_{top}. Is D_{top} greater than, equal to, or less that D_{noz}? Describe your reasoning on this result.

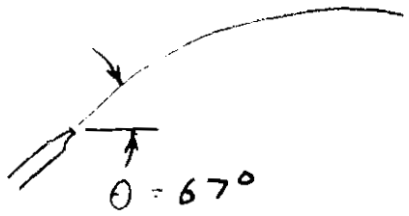
6) Work problem 5.91 as the other part of HW set 13.

FOUNTAIN EXPERIMENT - SOLUTION
FALL 2006

1.



$h = 6.5$ INCHES



2. EXPERIMENT 1 : $H_1 = 12''$ $H_2 = 14.25''$

EXPERIMENT 2 : $H_1 = 8''$ $H_2 = 11.5''$

3. CHOOSE DATUM AS SHOWN ABOVE. WRITE BERNOULLI EQN. FROM A TO B,

$$p_A^{\nearrow 0} + \frac{1}{2}\rho V_A^{2\nearrow 0} + \gamma z_A = p_B^{\nearrow 0} + \frac{1}{2}\rho V_B^2 + \gamma z_B$$

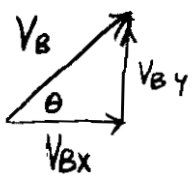
$$V_B = \sqrt{2g(z_A - z_B)} = \sqrt{2gH_1}$$

EXPERIMENT 1 : $V_B = 8.02 \frac{FT}{S}$

EXPERIMENT 2 : $V_B = 6.55 \frac{FT}{S}$

4. APPLY BERNOULLI EQN FROM B TO C,

$$p_B^{\nearrow 0} + \frac{1}{2}\rho V_B^2 + \gamma z_B = p_C^{\nearrow 0} + \frac{1}{2}\rho V_C^2 + \gamma z_C$$



$$V_B^2 = V_{Bx}^2 + V_{By}^2$$

V_C →

$$V_C = V_{Cx} = V_{Bx}$$

HORIZONTAL VELOCITY COMPONENT OF WATER STREAM DOES NOT CHANGE, WHY?

FOUNTAIN – CONTINUED

$$\gamma (z_c - z_B) = \frac{1}{2}\rho \left(V_B^2 - V_c^2 \right) \rightarrow V_{Bx}^2, \quad \gamma = \rho g$$

$$z_c - z_B = \frac{1}{2g}\left(V_{Bx}^2 + V_{By}^2 - V_{Bx}^2 \right)$$

$$z_c - z_B = \frac{1}{2g}\left(V_B \sin\theta \right)^2$$

EXPERIMENT 1: $z_c - z_B = 10.2$ IN. THEORY

$H_2 - h = 7.75$ IN. EXPERIMENT

EXPERIMENTAL RESULT IS 76% OF THEORETICAL.

EXPERIMENT 2: $z_c - z_B = 6.77$ IN. THEORY

$H_2 - h = 5.00$ IN. EXPERIMENT

EXPERIMENTAL RESULT IS 74% OF THEORETICAL.

THE THEORETICAL HEIGHT IS GREATER THAN THE MEASURED BECAUSE THE BERNOULLI EQN. NEGLECTS FRICTION. ALSO THE MEASUREMENT OF $\theta = 67°$ MAY BE OFF.

5. $V_B A_B = V_c A_c$ BUT $V_c = V_{Bx}$

$$\sqrt{V_{Bx}^2 + V_{By}^2}\, A_B = V_{Bx} A_c$$

$\Rightarrow A_c > A_B$ UNLESS $V_{By} = 0$ (HORIZONTAL STREAM)

$D_c > D_B$

D_{TOP} IS GREATER THAN D_{NOZ} BECAUSE THE VELOCITY IS LESS AT THE TOP THAN AT THE NOZZLE TIP.

5.91

GIVEN: $u = V(x^3 + xy^2)$

$v = V(y^3 + yx^2)$

$w = 0$

V IS A CONSTANT

FIND: DOES THIS FLOW FIELD SATISFY CONTINUITY?

SOLUTION:

DOES $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$?

$$\frac{\partial u}{\partial x} = V(3x^2 + y^2)$$

$$\frac{\partial v}{\partial y} = V(3y^2 + x^2)$$

$$\frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = V(3x^2 + y^2) + V(3y^2 + x^2) + 0$$

$$\neq 0$$

CONTINUITY IS NOT SATISFIED