6.39

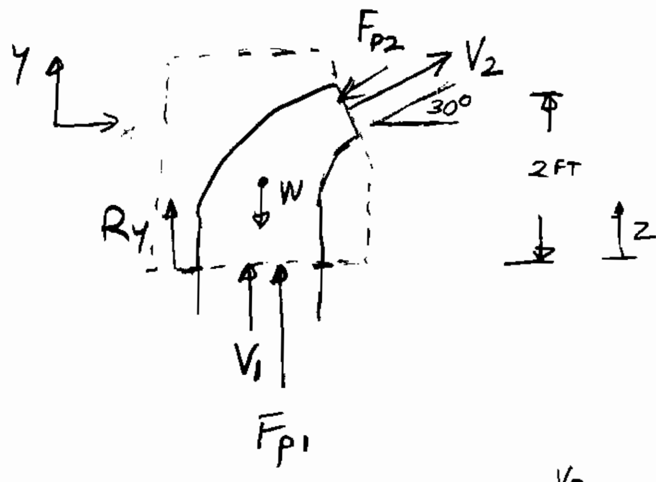
GIVEN: $\gamma = 62.4 \ LBf/FT^3$

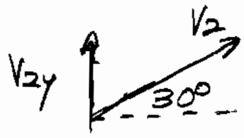
$V_2 = 125 \ FT/S \quad 130$

$W_N = 100 LBf$

$\forall_N = 1.8 \ FT^3$

$A_1 = 1.0 \ FT^2 \qquad A_2 = 0.5 FT^2$





FIND: VERTICAL FORCE AT FLANGE

SOLUTION: $\Sigma F_y = \frac{d}{dt}(\text{M}) + \dot{m}(V_{2y} - V_{1y})$

$$-W + p_1 A_1 + \cancel{F_{p2}}^{\,0} + R_y = 0 + \rho_2 V_2 A_2 \left(V_2 \sin 30^\circ - V_1\right)$$

$$W = W_n + W_{H_2O} = 100 LBf + (1.8 \ FT^3)\left(62.4 \frac{LBf}{FT^3}\right) = 212 \ LBf$$

USE BERNOULLI EQN. AND CONSERVATION OF MASS TO FIND p_1 AND V_1.

$$V_1 = \frac{A_2}{A_1} V_2 = \left(\frac{0.5}{1.0}\right) 130 = 65 \ FT/S$$

$$p_1 + \cancel{\gamma z_1}^{\,0} + \tfrac{1}{2}\rho V_1^2 = \cancel{p_2}^{\,0 \text{ GAGE}} + \gamma z_2 + \tfrac{1}{2}\rho V_2^2$$

$$p_1 = \gamma z_2 + \tfrac{1}{2}\rho\left(V_2^2 - V_1^2\right) = (62.4)(2) + \tfrac{1}{2}(1.94)\left(130^2 - 65^2\right)$$

$$p_1 = 12420 \ PSFG$$

$$R_y = 212 - (12420)(1.0) + (1.94)(100)(0.5)\left[\cancel{130(0.5) - 65}\right] \rightarrow 0$$

$$= -12210 \ LBf$$

$$\boxed{R_y = 12{,}210 \ LBf \text{ ACTING DOWNWARD}}$$

6.86 (1)

GIVEN: WATER ROCKET AS SHOWN.

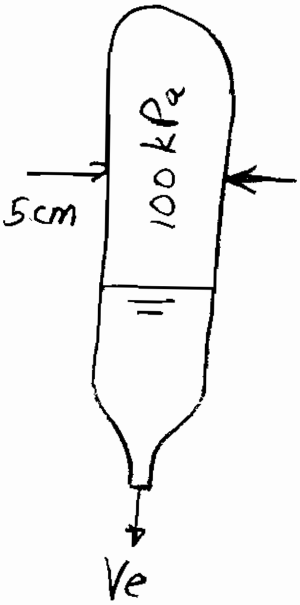
NEGLECT DRAG FORCE.

USE BERNOULLI EQN. TO CALCULATE EXIT VELOCITY OF WATER

$A_e = \frac{1}{10} A_R$

$m_R = 50\,g$

$m_w(0) = 100\,g$



FIND: MAXIMUM ROCKET VELOCITY

CONSERVATION OF MOMENTUM FOR ROCKET.

$$-\cancel{D} - W + (\cancel{p_e} - p_o)A_e + \rho V_e^2 A_e = m \frac{dV_R}{dt}$$

NEGLECT

$W \to m(t)\,g$

$\to 0$, $p_e = p_o$

$$-m(t)\,g + \rho V_e^2 A_e = m(t)\frac{dV_r}{dt}$$

ρ: WATER DENSITY

$$A_e = \frac{1}{10}\left(\frac{\pi}{4}\right)(.05\,m)^2$$

USE BERNOULLI EQN. TO FIND V_e.

$$\frac{\cancel{p_1}}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{\cancel{p_2}}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$100\,kPa/\gamma$

$p_2 = 0$ GAGE

ASSUME THAT AIR PRESSURE REMAINS CONSTANT

OKAY APPROXIMATION IF AIR VOLUME IS LARGE COMPARED TO WATER VOLUME.

6.86 (2)

NEGLECT: $z_2 - z_1$

SMALL COMPARED TO $\frac{p_1}{\gamma}$ TERM.

RESULT FROM BERNOULLI EQN. $V_2 = V_e = \sqrt{\frac{2p_1}{\rho}}$

BACK TO ROCKET EQUATION; $-m(t)g + 2p_1 A_e = m(t)\frac{dV_R}{dt}$

NEED TO FIND $m(t)$.

$$m(t) = m_R + m_w(t) = 50g + m_w(t)$$

USE CONSERVATION OF MASS TO FIND

$m_w(t) \rightarrow$ TANK DRAINING PROBLEM.

$$\frac{dm_w}{dt} + \underbrace{\dot{m}_{OUT}}_{\rho A_e V_e} - \underbrace{\cancel{\dot{m}_{IN}}}_{0} = 0$$

$$\frac{dm_w}{dt} = -\rho A_e V_e \, , \quad V_e = \sqrt{\frac{2p_1}{\rho}}$$

$$\frac{dm_w}{dt} = -A_e\sqrt{2p_1\rho}$$

$$m_w = -A_e\sqrt{2\rho p_1}\, t + C_1$$

$$m_w(0) = 100\,g \Rightarrow C_1 = 100\,g$$

$$m_w = m_w(0) - A_e\sqrt{2\rho p_1}\, t$$

BACK TO ROCKET EQN.

$$\frac{dV_R}{dt} = -g + 2p_1 A_e \frac{1}{m(t)}$$

$$\frac{dV_R}{dt} = -g + \frac{2p_1 A_e}{m_p + m_w(0) - A_e\sqrt{2\rho p_1}\,t}$$

LET $2p_1 A_e = C_1$, $m_p + m_w(0) = C_2$

AND $A_e\sqrt{2\rho p_1} = C_3$.

$$\frac{dV_R}{dt} = -g + \frac{C_1}{C_2 - C_3 t}$$

CONSTANT OF INTEGRATION

$$V_R = -gt + \int \frac{C_1}{C_2 - C_3 t}\,dt + C_4$$

$$V_R = -gt - \frac{C_1}{C_3}\ln(C_2 - C_3 t) + C_4$$

$$V_R(0) = 0 = -\frac{C_1}{C_3}\ln C_2 + C_4 \Rightarrow C_4 = \frac{C_1}{C_3}\ln C_2$$

$$V_R = -gt + \frac{C_1}{C_3}\left[-\ln(C_2 - C_3 t) + \ln C_2\right]$$

$$V_R = -gt + \frac{C_1}{C_3}\ln\left(\frac{C_2}{C_2 - C_3 t}\right)$$

V_{MAX} WILL OCCUR AT THE TIME WHEN THE ROCKET RUNS OUT OF WATER.

CALL THIS TIME t_{MAX}.

$$m_w = m_w(0) - A_e \sqrt{2\rho p_1}\, t$$

$$t_{MAX} = \frac{m_w(0) - \cancel{m_w}^{\,0}}{A_e \sqrt{2\rho\, p_1}}$$

$$t_{MAX} = \frac{0,1}{\frac{1}{10}\left(\frac{\pi}{4}\right)(.05)^2 \sqrt{2\,(1000)\,10^5}} = 0,0360\ S$$

$$V_{MAX} = -g\, t_{MAX} + \frac{c_1}{c_3} \ln\left(\frac{c_2}{c_2 - c_3 t_{MAX}}\right)$$

$$V_{MAX} = -0.35\ \frac{m}{S} + 15,49\ \frac{m}{S}$$

$$V_{MAX} = 15.1\ \frac{m}{S}$$