15.4

GIVEN: WATER FLOWS AT $12 m^3/s$ IN A 3 m WIDE CHANNEL.

FIND: FROUDE NUMBER AND TYPE OF FLOW FOR 30 cm, 1.0 m, AND 2.0 m DEPTHS. CRITICAL DEPTH.

SOLUTION:

$$Fr = \frac{V}{\sqrt{gy}} = \frac{Q}{A\sqrt{gy}} = \frac{Q}{wy\sqrt{gy}}$$

$y = 30$ cm $\quad Fr = \dfrac{12}{3(.3)\sqrt{(9.8)(.3)}} = 7.8$

$Fr = 7.8$ SUPER CRITICAL FLOW

$y = 1.0$ m $\quad Fr = 1.28$ SUPERCRITICAL FLOW

$y = 2.0$ m $\quad Fr = 0.45$ SUB CRITICAL FLOW

CRITICAL DEPTH $\quad Fr = 1 = \dfrac{Q}{wy_c\sqrt{gy_c}}$

$$y_c^{\frac{3}{2}} = \frac{Q}{w\sqrt{g}} \quad y_c = \left(\frac{Q}{w\sqrt{g}}\right)^{\frac{2}{3}}$$

$$\boxed{y_c = 1.18 \text{ m}}$$

15.11 GIVEN: LONG RECTANGULAR CHANNEL (4 m WIDE) HAS A FREE OUTFALL. THE DEPTH AT THE BRINK IS 0.35 m.

FIND: DISCHARGE IN CHANNEL

SOLUTION: AT THE BRINK OF A FREE OUTFALL, $h = 0.71 y_c$ (FIGURE 15.10)

$$\Rightarrow \quad y_c = \frac{.35 \text{ m}}{.71} = 0.493 \text{ m}$$

$$y_c = \left(\frac{q}{\sqrt{g}}\right)^{\frac{2}{3}} = \left(\frac{Q}{w\sqrt{g}}\right)^{\frac{2}{3}}$$

$$y_c^{\frac{3}{2}} = \frac{Q}{w\sqrt{g}}$$

$$Q = w\sqrt{g}\; y_c^{\frac{3}{2}}$$

$$Q = 4\sqrt{9.8}\,(.493)^{\frac{3}{2}}$$

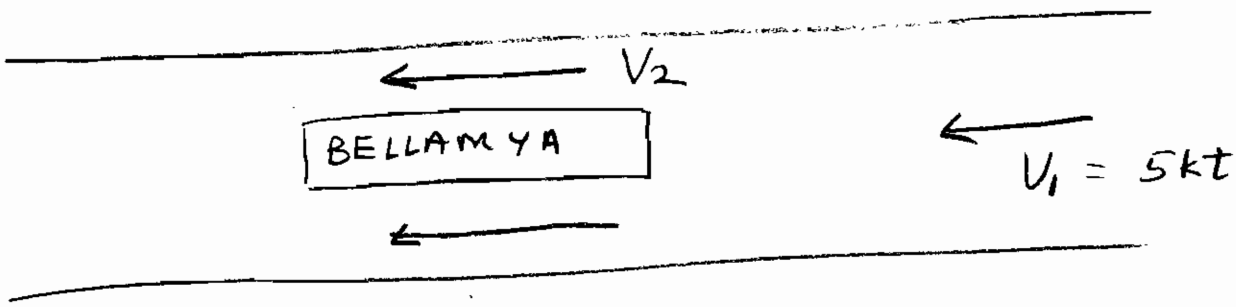
$$\boxed{Q = 4.33 \ \frac{m^3}{s}}$$

15.22 (1) GIVEN: SUPERTANKER "BELLAMYA"

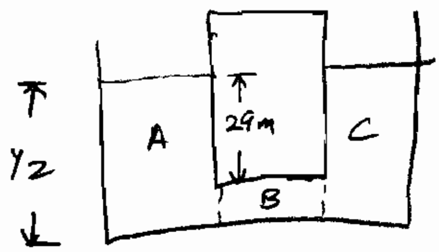
STEAMING AT 5 kt THROUGH A CHANNEL THAT IS 35 M DEEP AND 200 M WIDE. THE DRAFT OF THE BELLAMYA IS 29 M. ITS WIDTH AND LENGTH ARE 63 M AND 414 M.

FIND: THE LOWERING OF THE WATER AROUND THE BELAMYA.

SOLUTION: DO A COORDINATE TRANSFORMATION SO THAT THE BELLAMYA IS FIXED!



PLAN VIEW



END VIEW

$$A_2 = A_A + A_B + A_C$$

$$A_A + A_C = (200 - 63)\, y_2$$

$$A_B = (y_2 - 29)\, 63$$

$$A_2 = 137 y_2 + 63 y_2 - 1837$$

15.22 (2)

CONSERVATION OF MASS

$$V_1 A_1 = V_2 A_2$$

$$V_1 (35)(200) = V_2 (200 y_2 - 1837)$$

$$V_2 = V_1 \frac{7000}{(200 y_2 - 1837)}$$

CONSERVATION OF ENERGY

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$y_1 = 35\,m \qquad V_1 = 5\,kt = 2.575\,m/s$

$$35 + \frac{(2.575)^2}{2(9.8)} = y_2 + \frac{\left(\frac{18025}{200 y_2 - 1837}\right)^2}{2(9.8)}$$

$$35.338 = y_2 + \frac{1.6577\times10^7}{(200 y_2 - 1837)^2}$$

$$\underbrace{(200 y_2 - 1837)^2}\, 35.34 = y_2 (\quad)^2 + 1.6577\times10^7$$

$$(200 y_2 - 1837)^2 = 40000 y^2 - 734800 y + 3.374\times10^6$$

15.22 (3)

$$1.414\times10^{6}\,y_2^{2} - 2.597\times10^{7}\,y_2 + 1.193\times10^{8} =$$

$$40000\,y_2^{3} - 734800\,y_2^{2} + 3.374\times10^{6}\,y_2 + 1.6577\times10^{7}$$

COLLECT TERMS

$$40000\,y_2^{3} - 2.149\times10^{6}\,y_2^{2} + 2.934\times10^{7}\,y_2 - 1.027\times10^{8} = 0$$

Root solver to find water depth around the Bellamya

$g(y) := 40000y^{3} - 2149000y^{2} + 29340000\cdot y - 102700000$ (units of meters)

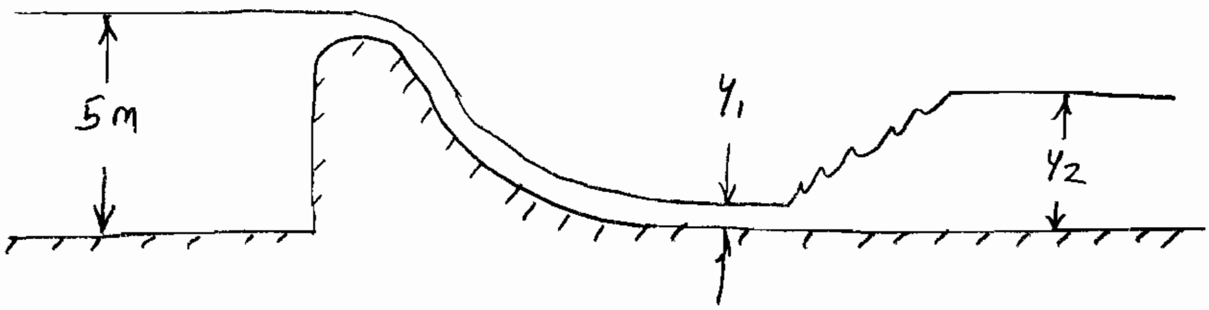
First guess: $y := 35$

$h := \mathrm{root}(g(y), y)$ $h = 34.74$

SHIP SQUAT = 35 m - 34.74 m

SHIP SQUAT = 0.26 m

15.29 (1) GIVEN: SPILLWAY AS SHOWN WITH $2.5\ m^3/s$ PER METER DISCHARGE.



FIND: y_2

SOLUTION: $$y_2 = \frac{y_1}{2}\left(\sqrt{1+8\,Fr_1^{\,2}} - 1\right)$$

NEED TO FIND y_1 AND Fr_1.

SUBCRITICAL FLOW UPSTREAM OF THE SPILLWAY. CALL THIS LOCATION "0".

$y_0 = 5\ m$, WIDTH $\sim$ $W = 1\ m$, $Q = 2.5\ \frac{m^3}{s}$

$$V_0 = \frac{Q}{A_0} = \frac{2.5}{5\ (1)} = 0.5\ \frac{m}{s}$$

THE SPECIFIC ENERGY OF THE FLOW IS: $E = y_0 + \frac{V_0^{\,2}}{2g}$

$$E = 5\,m + \frac{.5^2}{2(9.8)} = 5.013\ m$$

15.29 (2)

NOW: $y_1 + \frac{V_1^2}{2g} = 5.013$ m

CONTINUITY: $y_0 V_0 = y_1 V_1$

$$V_1 = \frac{y_0 V_0}{y_1} = \frac{5(.5)}{y_1} = \frac{2.5}{y_1}$$

$$y_1 + \frac{(2.5/y_1)^2}{2g} = 5.013$$

$$y_1 + \frac{0.3189}{y_1^2} = 5.013$$

$$y_1^3 - 5.013\, y_1^2 + 0.3189 = 0$$

Root solver to find the depth of the flow at the bottom of the spillway.

$g(y) := y^3 - 5.013y^2 + 0.3189$ First guess: $y := 6$ (units of feet)

$h := \text{root}(g(y), y)$ $h = 5.0002$

The first guess yields the original subcritical depth of 5 meters.

Second guess: $y := 1$

$h := \text{root}(g(y), y)$ $h = 0.2591$

The second guess yields the alternate depth of 0.259 meters. This is the ~~sub~~critical flow depth - the depth that occurs at the bottom of the spillway.
SUPER

$y_1 = 0.259$ m THIS IS THE FLOW DEPTH AT THE BOTTOM OF THE SPILLWAY

15.29 (3)

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}}$$

$$V_1 y_1 = V_0 y_0$$

$$V_1 = V_0 \frac{y_0}{y_1} = 0.5 \frac{m}{s} \left(\frac{5 m}{.259 m} \right) = 9.65 \frac{m}{s}$$

$$Fr_1 = \frac{9.65}{\sqrt{9.8(0.259)}} = 6.06$$

$$y_2 = \frac{.259}{2} \left[\sqrt{1 + 8(6.06)^2} - 1 \right]$$

$$\boxed{y_2 = 2.09 \text{ m}}$$