5.1

GIVEN: THE DISCHARGE OF WATER IN A 25 cm PIPE IS 0.04 m^3/s

FIND: MEAN VELOCITY

SOLUTION:

$$\bar{V} = \frac{Q}{A}$$

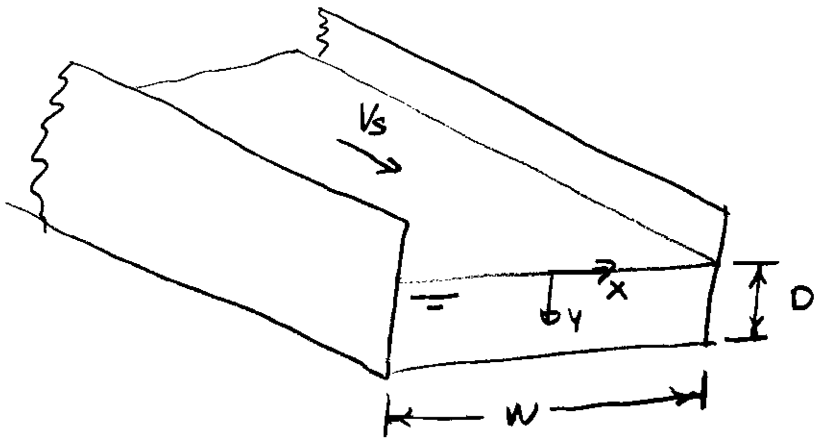
$$A = \frac{\pi}{4} D^2$$

$$\bar{V} = \frac{4Q}{\pi D^2}$$

$$\bar{V} = \frac{4\left(0.04\ \frac{m^3}{s}\right)}{\pi\ (0.25\,m)^2}$$

$$\bar{V} = 0.815\ \frac{m}{s}$$

5.9 GIVEN: WATER FLOW IN A TWO-DIMENSIONAL CHANNEL.



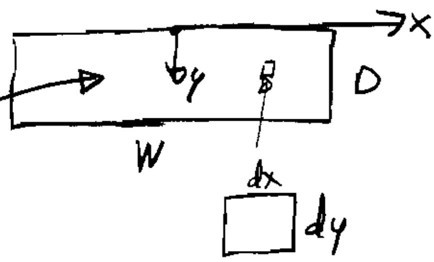
THE VELOCITY PROFILE IS GIVEN BY:

$$V(x,y) = V_s\left(1-\frac{4x^2}{W^2}\right)\left(1-\frac{y^2}{D^2}\right)$$

WHERE V_s IS THE VELOCITY AT THE WATER SURFACE MIDWAY BETWEEN THE CHANNEL WALLS.

FIND: THE DISCHARGE (VOLUME FLOW RATE)

$$Q = \int_A V\,dA$$



A ~ AREA OF INTEGRATION

$$dA = dx\,dy$$

$$Q = \int_{y=0}^{y=D}\int_{x=-W/2}^{x=W/2} V_s\left(1-\frac{4x^2}{W^2}\right)\left(1-\frac{y^2}{D^2}\right)dx\,dy$$

$$Q = V_s \int_{y=0}^{y=D} \left(1 - \frac{y^2}{D^2}\right) \left[x - \frac{4}{3}\frac{x^3}{W^2} \right]_{-\frac{W}{2}}^{\frac{W}{2}} dy$$

$$Q = V_s \int_{y=0}^{y=D} \left(1 - \frac{y^2}{D^2}\right) \left[\frac{W}{2} - \left(-\frac{W}{2}\right) - \frac{4}{3}\left(\frac{W}{8}\right) + \frac{4}{3}\left(-\frac{W}{8}\right) \right] d$$

$$Q = V_s \left[y - \frac{1}{3}\frac{y^3}{D^2} \right]_0^D \left[W - \frac{W}{3} \right]$$

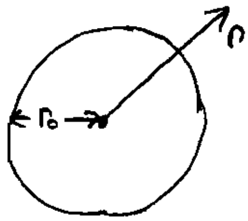
$$Q = V_s \left[D - \frac{1}{3} D \right] \frac{2}{3} W$$

$$\boxed{Q = \frac{4}{9} V_s D W}$$

5.21

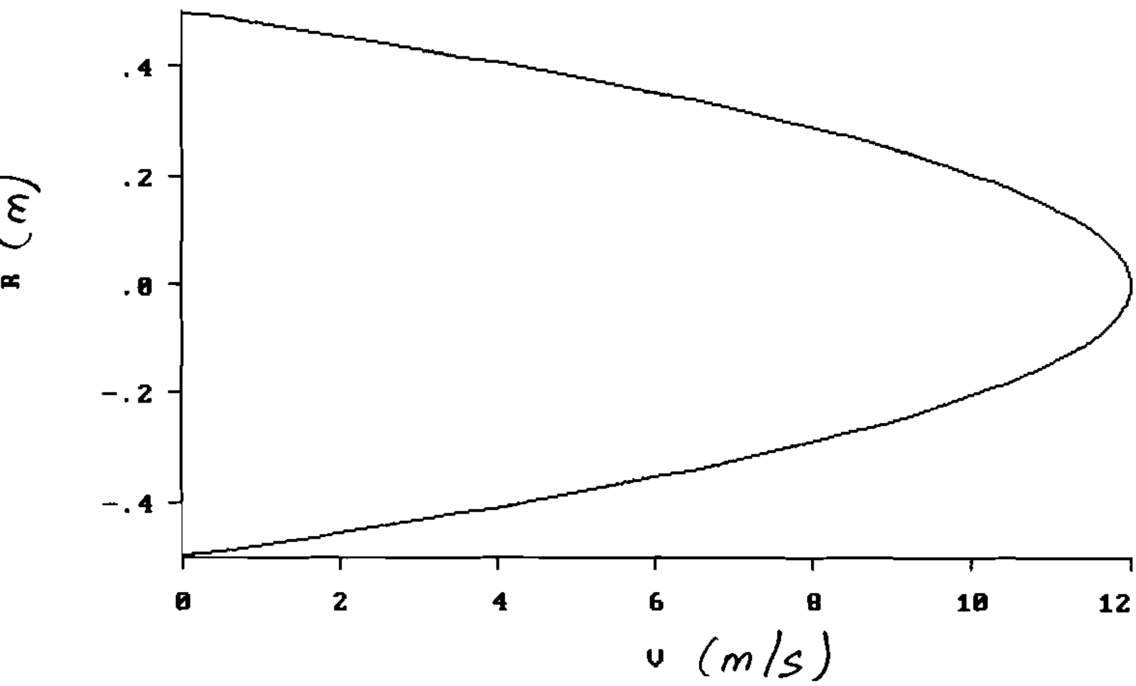
GIVEN: 1 m DIAMETER ROUND PIPE WITH VELOCITY DISTRIBUTION: $V = 12(1 - r^2/r_0^2)$ m/s.

FIND: MEAN VELOCITY
DISCHARGE
PLOT VELOCITY DISTRIBUTION



$r_0 = 0.5$ m

SOLUTION:



$$Q = \int_A \underset{\sim}{V} \cdot \underset{\sim}{dA} = \int_0^{r_0} V\, 2\pi r\, dr \qquad \bar{V} = \frac{Q}{A}$$

$$Q = \left(12 \tfrac{m}{s}\right)(2\pi) \int_0^{r_0} \left(r - \frac{r^3}{r_0^2}\right) dr$$

$$Q = 24\pi \tfrac{m}{s} \left[\tfrac{1}{2} r^2 - \tfrac{1}{4} \frac{r^4}{r_0^2}\right]_0^{r_0} = 24\pi \tfrac{m}{s} \left[\tfrac{1}{2} r_0^2 - \tfrac{1}{4} r_0^2\right]$$

$$Q = 6\pi \tfrac{m}{s}\, r_0^2 \qquad \bar{V} = \frac{6\pi \frac{m}{s}\, r_0^2}{\pi r_0^2}$$

$$\boxed{Q = 4.71 \tfrac{m^3}{s}} \qquad \boxed{\bar{V} = 6 \tfrac{m}{s}}$$