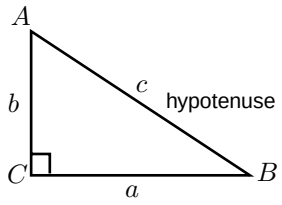
Pythagoras' theorem

Introduction

Pythagoras' theorem relates the lengths of the sides of a right-angled triangle. This leaflet reminds you of the theorem and provides some revision examples and exercises.

1. Pythagoras' theorem

Study the right-angled triangle shown.

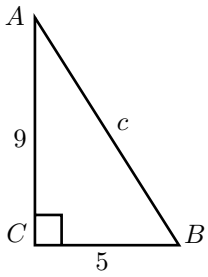


In any right-angled triangle, ABC, the side opposite the right-angle is called the **hypotenuse**. Here we use the convention that the side opposite angle A is labelled a. The side opposite B is labelled b and the side opposite C is labelled c.

Pythagoras' theorem states that the square of the hypotenuse, (c^2), is equal to the sum of the squares of the other two sides, $(a^2 + b^2)$.

Pythagoras' theorem: $c^2 = a^2 + b^2$

Example



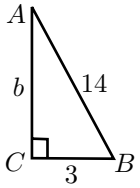
Suppose $AC = 9$cm and $BC = 5$cm as shown. Find the length of the hypotenuse, AB.

Solution
Here, $a = BC = 5$, and $b = AC = 9$. Using the theorem

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 9^2 \\ &= 25 + 81 \\ &= 106 \\ c &= \sqrt{106} = 10.30 \qquad \text{(2dp.)} \end{aligned}$$

The hypotenuse has length 10.30cm.

Example
In triangle ABC shown, suppose that the length of the hypotenuse is 14cm and that $a = BC =$ 3cm. Find the length of AC.



Solution
Here $a = BC = 3$, and $c = AB = 14$. Using the theorem

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 14^2 &= 3^2 + b^2 \\ 196 &= 9 + b^2 \\ b^2 &= 196 - 9 \\ &= 187 \\ b &= \sqrt{187} = 13.67 \qquad \text{(2dp.)} \end{aligned}$$

The length of AC is 13.67cm.

Exercises
1. In triangle ABC in which $C = 90°$, $AB = 25$ cm and $AC = 17$ cm. Find the length BC.
2. In triangle ABC, the angle at B is the right-angle. If $AB = BC = 5$ cm find AC.
3. In triangle CDE the right-angle is E. If $CD = 55$cm and $DE = 37$cm find EC.

Answers
1. 18.33 cm. (2dp.)
2. $AC = \sqrt{50} = 7.07$ cm. (2dp.)
3. $EC = \sqrt{1656} = 40.69$ cm. (2dp.)