FOR 274: Forest Measurement and Inventory

Combining Errors Handout

Tip: **ALWAYS** calculate the results without the error and then calculate the error & **ALWAYS** present results with units: e.g., Biomass = 300 ± 20 kg

For More Information and More Rules/Examples see:
Pentz, M. and Shott, M. Handling Experimental Data, Open University Press.

If measurements are independent, then the total error, $\pm E_T$ associated with two measurements A ($\pm E_A$) and B ($\pm E_B$) is calculated using the following rules (These rules are of the same form for greater than 2 measurements):

1. Combining Errors in a Sum of Difference

If Total = Measure A + Measure B or Total = Measure A – Measure B Then:

$$\mathbf{E_T = \sqrt{[(E_A)^2 + (E_B)^2]}}$$

This rule is often referred to as "the square root of the sum of the squares". The total error E_T will **always** be greater than each of the component errors, but will **always** be less than the sum of those errors.

For 3 measurements: A, B, & C the equation for E_T would be:

$$\mathbf{E_T = \sqrt{[(E_A)^2 + (E_B)^2 + (E_C)^2]}}$$

2. Combining Errors in a Product or a Ratio

If Total = Measure A* Measure B or Total = Measure A / Measure B Then:

$$\mathbf{E_T / Total = \sqrt{[(E_A/Measure\ A)^2 + (E_B/Measure\ B)^2]}}$$

Therefore, once you have calculated T, you calculate E_T by multiplying T by the result of this equation.

For 4 measurements: A, B, & C the equation for E_T would be:

$$\mathbf{E_T / Total = \sqrt{[(E_A/Measure\ A)^2 + (E_B/Measure\ B)^2 + (E_C/Measure\ C)^2]}}$$

3. Combining Errors in a Power Law

If $T = A^n$ Then:

$$\mathbf{E_T / T = n * E_A / A}$$

Worked Through Example 1: Sum and Difference

A forester measures the volume of two log sections that used to be part of the same tree. The forester wants to know the total volume of the logs combined. The measures are:

Measure A = 50 board feet ± 2 board feet
Measure B = 120 board feet ± 10 board feet

The Total Volume = Measure A + Measure B = 120 + 50 = 170 Board Feet

The Error is calculated using:

$E_T = \sqrt{[(E_A)^2 + (E_B)^2]}$ board feet

$E_T = \sqrt{[(2)^2 + (10)^2]}$ board feet

$E_T = \sqrt{[(4) + (100)]}$ board feet

$E_T = \sqrt{[104]}$ board feet

E_T = 10.2 board feet

Therefore the Total Measure of log volume is **<u>170 ± 10.2 board feet</u>**

Worked Through Example 2: Product

A forester wants to calculate the volume of a cylindrical log based on the measurement of stem diameter and log length.

Stem Diameter = 12 inches ± 0.5 inches
Log Length = 16 feet ± 0.5 feet

Volume = π* (Diameter/2)^2 * Log Length

π* (Diameter/2)^2 = 3.14159 * (6)^2 = 113.09 sq inches

Volume (cubic inches) = 113.09 * (16*12) = 21713.28 cubic inches

E_T/ Total = $\sqrt{[(E_A/\textbf{Measure A})^2 + (E_B/\textbf{ Measure B})^2]}$
E_T/ 21713.28 = $\sqrt{[(0.5/\textbf{12})^2 + (0.5/\textbf{ 16})^2]}$
E_T/ 21713.28 = $\sqrt{[(0.042)^2 + (0.03125)^2]}$
E_T/ 21713.28 = $\sqrt{[(0.002) + (0.00098)]}$ = **0.05**

E_T = 21713.28 * 0.05 = 1085.664

Therefore the Total Measure of log volume is **<u>21713.28 ± 1085.664 cubic inches</u>**