Using Mathematics in Forestry[1]

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Most jobs require good mathematics skills, and forestry is no exception. The exercises in this handout will show you some of the ways that foresters use mathematics in their work.

Foresters usually deal with large land areas. They need to know what kind of trees grow on their area, how many of them there are, and how big they are. Measuring every tree on a national forest covering one and half million acres and containing over 500 million trees would not only be time consuming and expensive, but probably impossible!

For these and other reasons, foresters only measure part of the forest, a process called sampling. They assume that the measured (sampled) part represents the whole area. Data collected from the sampled part is expanded to provide information for the unsampled portion.

Suppose we need to know how much usable timber is present in a 100-acre ponderosa pine stand. The entire stand (all 100 acres) is our population, but we can't measure every tree because it would take too much time. So the most efficient way to inventory the ponderosa pine timber is to sample it, which we'll do by establishing plots.

Statistics indicates that 20 plots of ¼-acre each will provide reliable information about the ponderosa pine timber. How can we figure out the plot size? By using some mathematical formulas, of course! Previous sampling experience shows that circular plots are easier to use than square ones.

To lay out a plot in the shape of a circle, we must know its radius, which we'll figure out using this formula:

$$\text{Area} = \text{Pi} \times \text{Radius}^2$$

> Note: if you don't know what pi is, here is a short description. Pi is the number you get when you divide the distance around a circle (its circumference) by the distance through its middle (the diameter). The distance around the outside of every circle is about three times the distance across it. But it's the "about" part that creates the puzzle of pi. Mathematicians call pi an irrational number because when you divide a circle's circumference by its diameter, the answer comes out in decimals that go on forever without any apparent pattern. Pi begins as 3.14159265, but it never ends. In 1999, a Japanese scientist used a supercomputer to calculate pi to about 206 billion digits, and it still goes on from there. All those digits aren't really necessary to use pi, of course – using only the first ten decimals, you can measure the earth's circumference to within a fraction of an inch. Pi is often shown in textbooks or in formulas using this Greek symbol: π

[1] This paper was prepared in 1990 for Mr. Kevin Steinmetz's sixth grade class at Humbolt Elementary School in John Day, Oregon. It has been used several times since then with other school groups.

The total area of an acre is 43,560 square feet. Since we want to use a ¼-acre plot, its area is 43,560 ÷ 4 or 10,890 square feet. We now have enough information to use the formula above to figure out our plot radius.

$$\text{Area} = \text{Pi} \times \text{Radius}^2$$

$$10{,}890 \text{ square feet} = 3.1416 \times \text{Radius}^2$$

$$10{,}890 \text{ square feet} \div 3.1416 = \text{Radius}^2$$

$$3{,}466.39 \text{ square feet} = \text{Radius}^2$$

$$\sqrt{3{,}466.39} \text{ square feet} = \text{Radius}$$

$$58.9 \text{ feet} = \text{Radius}$$

Each ¼-acre plot will be a circle with a radius of 58.9 feet. We'll use a wooden stake as the plot center, and mark the plot boundaries by measuring out a radius in several directions from the stake.

Great! We've just used our knowledge about the geometry of a circle to design plots that will help us answer our inventory question about the ponderosa pine timber.

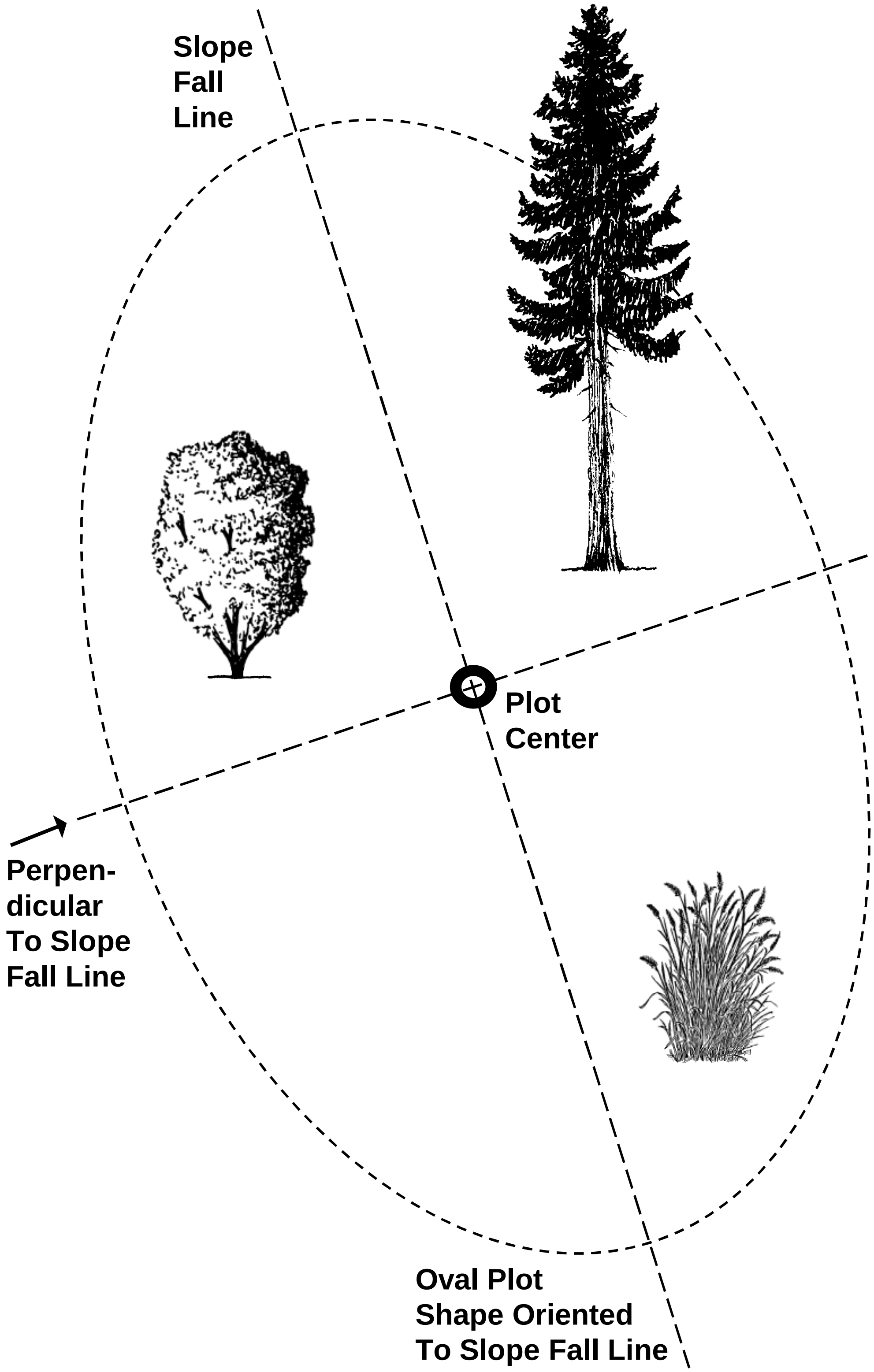
<u>Exercise 1</u>: If our circular plot has a radius of 58.9 feet, what is its diameter? [Do you remember the relationship between a circle's radius and its diameter?]

Diameter is: ____________________ feet.

<u>Exercise 2</u>: What would be the area (in square feet) and the radius (in feet) for circular plots of the following sizes? [Don't forget that 1 acre contains 43,560 square feet.]

Plot Size (Acres)	**Plot Area (Square Feet)**	**Plot Radius (Feet)**
1/4	10,890	58.9
1/5	____________	____________
1/10	____________	____________
1/20	____________	____________
1/50	____________	____________
1/100	____________	____________
1/250	____________	____________
1/300	____________	____________
1/500	____________	____________
1/1000	____________	____________

If the ground is absolutely flat, then the plot is a perfect circle with a radius as it was calculated above. The problems start when you are working on a hill because the plot's radius then varies, depending on the steepness of the hill. By projecting a plot's radius on a steep hillside, we see that it becomes oval in shape, not circular:



On sloping ground, plots have an oval shape with their long axis parallel to the slope (called the "slope fall line"). Note that a line perpendicular to the slope forms a "right angle" with the slope fall line. Plots on sloping ground need to have their radius adjusted using a factor that converts slope distance to what is called horizontal distance.

How do we adjust the radius of a plot that occurs on sloping ground? Well, you could figure out the adjusted radius using trigonometry (secants), but most foresters just carry around something called a slope correction table.

Here is part of the slope correction table for slopes ranging up to 61 percent:

Table 1: Slope correction factors

Slope Percent	Correction Factor
0 – 9	1.00
10 – 17	1.01
18 – 22	1.02
23 – 26	1.03
27 – 30	1.04
31 – 33	1.05
34 – 36	1.06
37 – 39	1.07
40 – 42	1.08
43 – 44	1.09
45 – 47	1.10
48 – 49	1.11
50 – 51	1.12
52 – 53	1.13
54 – 55	1.14
56 – 57	1.15
58 – 59	1.16
60 – 61	1.17

Now, let's use table 1's slope correction factors to figure out if some trees near our plot edge are "in or out" (inside or outside the plot radius). These trees near the plot edge are referred to as *borderline* trees. Here is a way to measure borderline trees:

1. Use an instrument called a clinometer to measure the slope percent from the center (side) of a borderline tree to the plot center (let's say that it is 30 percent).
2. Find a slope correction factor in table 1 for the slope percent you just measured (the correction factor is 1.04 for 30 percent).
3. Multiply the correction factor by the plot radius. This is called the corrected radius. For our ¼-acre plots, the result is: 58.88 feet × 1.04 = 61.24 feet.
4. Measure the slope distance from the center (side) of the tree to the plot center. If the measured distance is less than the corrected radius (61.24 feet in this example), then the tree is in; if the measured distance is more, the tree is out.

Exercise 3: Are the following trees on a ¼-acre plot in or out? Record the slope correction factor (from table 1) and corrected plot radius for each tree too.

Tree Number	Slope Percent From Tree to Plot Center	Slope Correction Factor	Corrected Plot Radius (Feet)	Distance From Tree to Plot Center (Feet)	In or Out?
1	20	___________	___________	59.1	______
2	25	___________	___________	58.6	______
3	30	1.04	61.24	62.9	Out
4	35	___________	___________	62.4	______
5	45	___________	___________	63.7	______
6	50	___________	___________	59.9	______
7	60	___________	___________	60.3	______

MEASURING CIRCUMFERENCE AND DIAMETER

Now that we know how to figure out which trees are in or out of the plot, we need to learn how to measure the size of trees that are in on the plot. First, we need to find out how big around each tree is.

There are two main ways we can describe the size of round objects like tree stems:

- We can measure their circumference, which is the distance around the outside of the trunk, or
- We can measure the circumference and convert it to diameter, which is the distance through the middle of a tree's trunk.

Foresters use special measuring tapes that show a tree's circumference on one side, and its equivalent diameter on the other side. How is that done? Actually, it's easy to do because circumference and diameter are closely related:

$$\text{Circumference} = \text{Pi} \times \text{Diameter}$$

$$\text{Diameter} = \text{Circumference} \div \text{Pi}$$

[Have you noticed yet that most mathematical formulas pertaining to circles use pi?]

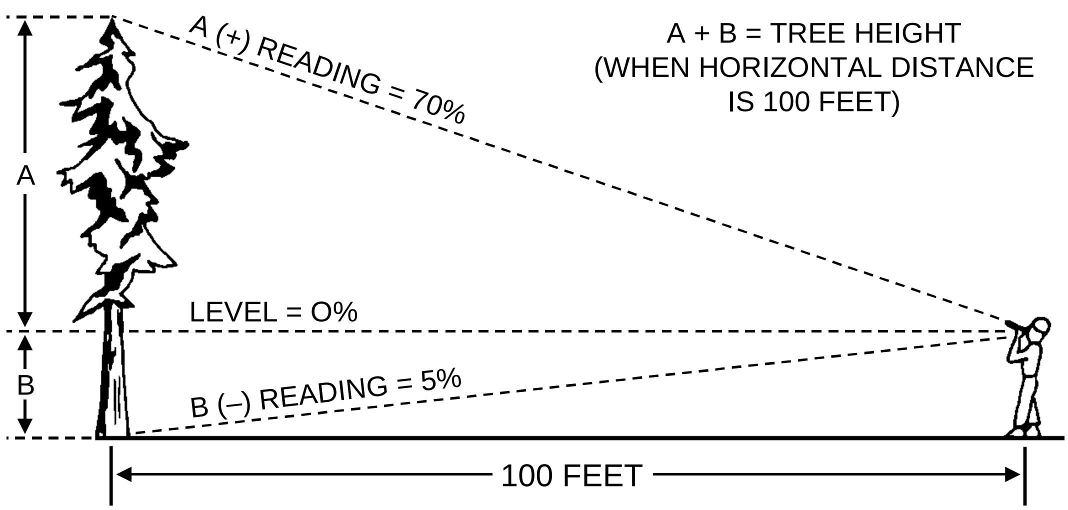
Exercise 4: What are the diameters or circumferences of the following trees? (You might be interested to know that these are actual measurements for some of the biggest trees found in the Blue Mountains of northeastern Oregon.)

Species	Circumference (Inches)	Diameter (Inches)
Subalpine Fir	132	________
Western Juniper	151	________
Lodgepole Pine	________	34.7
Ponderosa Pine	242	________
Engelmann Spruce	________	63.3
Grand Fir	218	________
Quaking Aspen	129	________
Douglas-fir	________	65.9

MEASURING TREE HEIGHTS

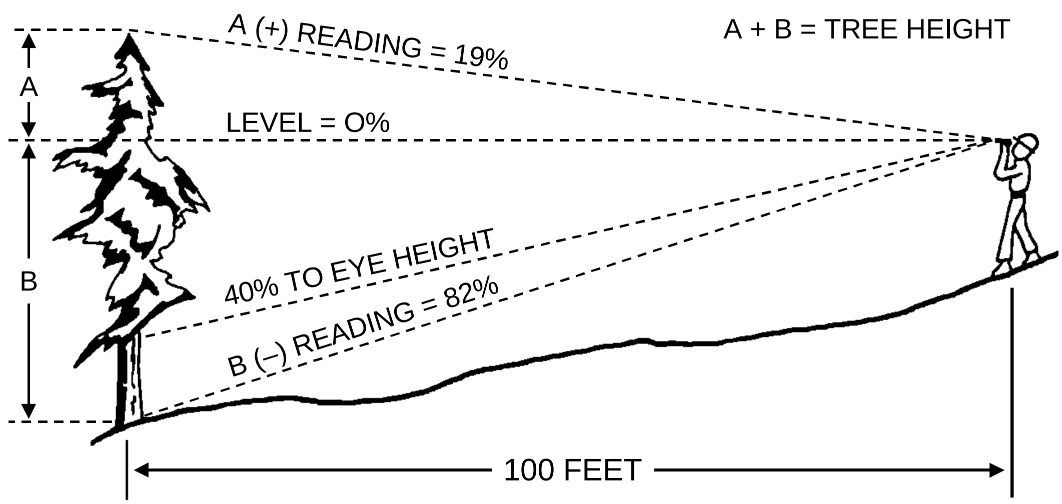
After measuring a tree's diameter or circumference, we then need to find out how tall it is. This can be tricky until you get the hang of it, especially when working on hillsides or sloping ground.

Foresters measure heights using an instrument called a clinometer (short for an "incline meter"). Many clinometers are based on percent, which means they are designed to be used at a distance of 100 feet from the tree. Let's look at an example on flat ground:



The A (+) readings on a clinometer mean that you are looking up; the B (–) readings mean that you are looking down. A level (not up or down) reading would be 0 on a clinometer. In our example above, the A (70) reading was looking up at the top of the tree; the B (5) reading was looking down at its base (where the trunk meets the ground). To get the tree's total height, you add the top reading (70) and the bottom reading (5) together: 75 feet tall for our example tree.

Now, let's look at a situation where measuring tree height is a little more complicated. When using a clinometer on sloping ground (hillsides), you must apply a slope correction factor just like we did when checking whether borderline trees were in or out of the plot area. Here's a height measurement example for sloping ground:



Here is the process you'd use to measure this tree's height:

1. Use the clinometer to measure the slope percent from you to your "eye height" on the tree trunk (this sighting is a line that's parallel to the ground surface). In our example, the slope percent is 40.
2. Find a slope correction factor in table 1 for the slope percent you just measured (1.08 is the slope correction factor for 40 percent).

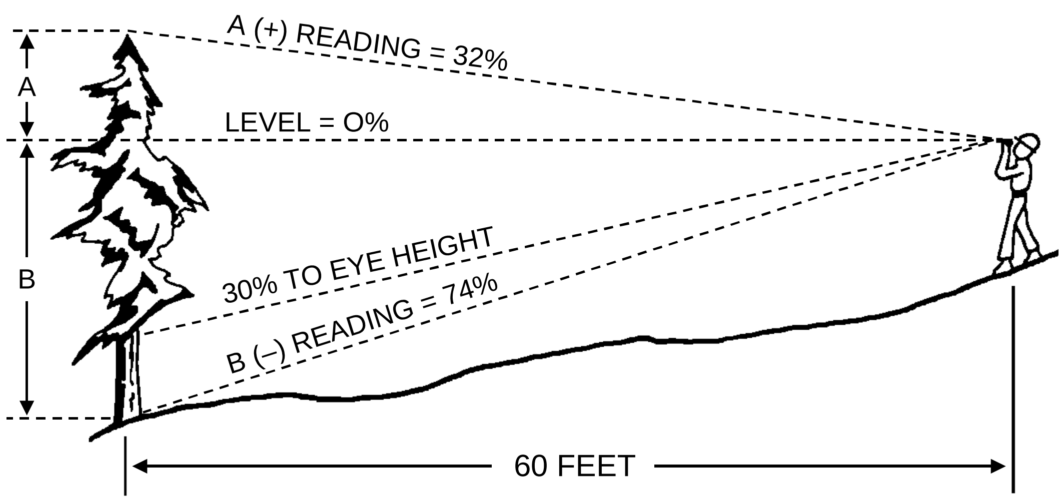
> Note: many clinometers show slope correction factors on the same scale as the slope percent values, which is handy because then you don't have to look them up in a table.

3. Multiply the slope correction factor by the baseline distance. In our example with a 100-foot baseline, the result is: 100 feet × 1.08 = 108 feet. This means that on a 40 percent slope, you must be 108 feet away from a tree to get the same answer as if you were 100 feet away on flat ground.
4. Back up to 108 feet from the tree before taking clinometer readings of its top and base.
5. Take clinometer readings of the tree's top and base, and add them together to come up with a total height. In our example, the result would be: 19 + 82 = 101 feet.

Exercise 5: What are the heights of the following trees?

Tree	Slope Percent to Tree	Slope Correction Factor	Corrected Baseline (Feet)	Clinometer Readings: Up (A)	Clinometer Readings: Down (B)	Total Height (Feet)
1	35	________	________	96	32	________
2	40	1.08	108	19	82	101
3	45	________	________	43	8	________
4	50	________	________	78	16	________
5	55	________	________	91	27	________
6	60	________	________	66	35	________

Next we'll look at another situation where measuring tree height is more complicated than usual. It is often hard to measure heights from 100 feet away if thick brush or downed logs hide your view of the tree's base. In those cases, you'll need to measure height from someplace other than 100 feet away and adjust the clinometer results accordingly. Here is our example tree:



Here is what you'd do to measure this tree's height:

1. Use the clinometer to measure a slope percent to eye height on the tree (30 percent in this example).
2. Get a slope correction factor from table 1 (1.04 for 30 percent).
3. Multiply the slope correction factor by your baseline distance: 60 feet × 1.04 = 62.4 feet.
4. Back up to 62.4 feet before taking clinometer readings of the tree's top and base.
5. Take clinometer readings of the top and base, and add them together. In our example, the result is: 32 + 74 = 106.

   Does this mean that the tree is 106 feet tall? No, it does not! Do you remember that a clinometer is designed to be used with a 100-foot baseline (see page 6)? Any time that you are not 100 feet away from the tree (slope-corrected distance), the clinometer readings must be adjusted to account for a different baseline distance. In our example, the slope-corrected baseline distance was only 60 feet, not 100.

6. Calculate a baseline adjustment factor by dividing your baseline distance by 100: 60 feet ÷ 100 = 0.6.
7. Multiply the sum of the clinometer readings (see step 5 above) by the adjustment factor to finally get a total height for your tree: 106 × 0.6 = 63.6 feet.

Exercise 6: What are the heights of the following trees?

Tree	**Baseline Distance (Feet)**	**Baseline Adjustment Factor**	**Clinometer Readings: Up (A)**	**Clinometer Readings: Down (B)**	**Total Height (Feet)**
1	40	________	89	47	_______
2	50	________	73	26	_______
3	60	.6	32	74	63.6
4	65	________	68	29	_______
5	75	________	52	11	_______
6	90	________	64	26	_______

Now, let's pull some of our information together. We began sampling the 100-acre ponderosa pine stand last week and three plots are done so far. Here is the data at this point:

PLOT 1 (¼ acre)

Tree	**Species**	**Diameter (Inches)**	**Height (Feet)**	**Age (Years)**
1	Ponderosa Pine	32.6	109	196
2	Ponderosa Pine	27.4	101	173
3	Grand Fir	18.9	88	93
4	Douglas-fir	29.4	106	204
5	Grand Fir	12.3	72	79
6	Quaking Aspen	8.6	66	58
7	Western Larch	14.2	91	87
8	Ponderosa Pine	9.6	48	54
9	Ponderosa Pine	15.2	68	74
10	Western Larch	16.9	96	86
	Average	________	________	________

How many trees per acre? ________

PLOT 2 (¼ acre)

Tree	Species	Diameter (Inches)	Height (Feet)	Age (Years)
1	Ponderosa Pine	33.9	111	194
2	Ponderosa Pine	30.7	108	188
3	Douglas-fir	26.9	98	147
4	Ponderosa Pine	27.8	102	156
5	Douglas-fir	21.6	92	106
6	Ponderosa Pine	28.9	103	129
	Average	________	________	________

How many trees per acre? ________

PLOT 3 (¼ acre)

Tree	Species	Diameter (Inches)	Height (Feet)	Age (Years)
1	Ponderosa Pine	26.2	98	114
2	Grand Fir	22.9	95	102
3	Ponderosa Pine	36.8	109	149
4	Douglas-fir	42.9	113	196
5	Lodgepole Pine	12.6	74	82
6	Ponderosa Pine	28.4	100	121
7	Ponderosa Pine	22.9	91	106
8	Grand Fir	18.7	89	97
9	Douglas-fir	27.6	99	112
	Average	________	________	________

How many trees per acre? ________

<u>Exercise 7</u>: Here's what you need to do now:

1. Calculate an average diameter, height, and age for each of the 3 plots, and record it in the tables (calculate an arithmetic average by summing the values in each column and then dividing by the total number of sample trees).
2. Calculate the number of trees per acre that each plot represents, and record your answers in the tables.

   How can you do this? Remember that each plot samples exactly ¼ of an acre. If you count the number of trees that are "in" on your plot, and then multiply by 4 to expand the sample to a whole acre, you will know how many trees per acre are represented by your sample.

> <u>Note</u>: when you sample (measure) a fraction of an acre, and then later want to expand the sample data so it represents a whole acre, then the sample values must be multiplied by the denominator of the plot size. For a 1/5-acre plot, each sample tree represents 5 trees (the denominator value); for a 1/20-acre plot, each sample tree represents 20 trees, etc.

3. Calculate an average for the 3 plots combined by filling in the table below:

Plot	Average Diameter (Inches)	Average Height (Feet)	Average Age (Years)	Trees Per Acre
1	_________	_________	_________	_________
2	_________	_________	_________	_________
3	_________	_________	_________	_________
Stand Average	_________	_________	_________	_________

4. What proportion of the sample trees, on the 3 plots combined, were ponderosa pines (don't forget that we are interested in how much ponderosa pine timber we have on the 100-acre tract)? __________ percent.

5. Fill in the blanks in this summary of our results so far. "After completing three plots, we can now say that the sampled area has an average of __________ trees per acre, with an average diameter of __________ inches, an average height of __________ feet, and an average age of __________ years. The proportion of ponderosa pines on the 3 plots was __________ percent."

Since this summary is for 3 plots only, the data will almost certainly change after all 20 samples are completed!

LET'S SUMMARIZE

What have you learned from this exercise?

- How to figure out the radius of a circular plot after being given its area.
- How to adjust a plot radius to account for sloping ground.
- That many mathematical formulas involving circles use pi, a special constant that we round off to 3.1416.
- How to measure the circumference and diameter of trees.
- How to use a clinometer to measure tree heights on either flat or sloping ground.