Final Exam

1. For the following statements, write T or F to indicate that the statement is true or false. (2 pts each)

(a) $1^\infty$ is one of the indeterminate forms. ______

(b) \[
\frac{2}{x(x^2 - 6x + 9)} = \frac{A}{x} + \frac{Bx + C}{x^2 - 6x + 9}
\]
for some real number $A$, $B$ and $C$. ______

(c) \[
\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}
\]
is an example of $p$-series. ______

(d) If $\lim_{k \to \infty} a_k = 0$, then $\sum_{k=1}^{\infty} a_k$ converges. ______

(e) $\cosh^2(x) - \sinh^2(x) = 1$. ______

2. Answer the following questions. (5 pts each)

(a) Evaluate $\lim_{z \to \infty} \left( 1 + \frac{10}{z^2} \right)^{z^2}$. 

(b) Evaluate the derivative of $f(x) = \tan^{-1}(\sqrt{x})$. 

3. Evaluate the following integrals. (5 pts each)

(a) \( \int x \cos x \, dx \)

(b) \( \int \frac{dx}{(16 - x^2)^{3/2}} \)

(c) \( \int \frac{dx}{x^2 + x - 2} \)

4. Determine whether the integral \( \int_0^\infty e^{-2x} \, dx \) converges or diverges. If it converges, evaluate it. (5 pts)
5. Determine if the following series converge/diverge. **Explain your reasoning such as the test you are using.** (5 pts each)

   (a) \[ \sum_{k=1}^{\infty} \frac{1}{k^2 + 1} \]

   (b) \[ \sum_{k=1}^{\infty} \frac{(-1)^k 5^k}{k!} \]

6. Find the Taylor series for \( f(x) = \cos x \) centered at \( a = -\pi \). **Write your answer using summation notation.** (10 pts)
7. Given that $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$ for $|x| < 1$, answer the following questions.

(a) Find the Maclaurin series of $f(x) = \frac{1}{1+x}$. (5pts)

(b) Find the Maclaurin series of $g(x) = \ln(1 + x)$. (5pts)

(c) Use the Ratio Test to determine the interval of convergence of the Maclaurin series that you obtained for the part (b). (10pts)
8. Graph the polar equation $r = 1 + \sin \theta$. (5 pts)

9. Evaluate the derivative of $f(x) = \sinh \sqrt{x^2 + 1}$. (5 pts)

10. For the following parametric equation, compute the arc length for $0 \leq t \leq \pi$. (10 pts)

\[ x = \frac{1}{2} t^2, \quad y = \frac{1}{3} t^3; \quad 0 \leq t \leq 2\sqrt{2} \]