Final Exam
Math 175, Calculus II, Spring 2012

Section: 01 Name: ____________________________

This test is closed book and closed notes. Calculators of any kind are not allowed. You must clearly show your work to receive credit. Unless otherwise stated, you do not need to simplify your answer.

Answer 15 problems. If you answer more than 15 problems, your grade for the final will be based off of your 15 best answers.

1. Let \( f(t) = \tan^{-1}(e^t) \). Find \( f'(t) \).

   ANSWER: \( f'(t) = \frac{e^t}{1 + e^{2t}} \).

2. Find \( \int x^2 \sin x \, dx \).

   ANSWER: \( \int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x - 2 \cos x + C \).
3. Find \( \lim_{x \to \infty} \left( 1 + \frac{1}{x} \right)^{2x} \).

ANSWER: \( \lim_{x \to \infty} \left( 1 + \frac{1}{x} \right)^{2x} = e^2 \).

4. Find \( \int x \ln x \, dx \).

ANSWER: \( \int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C \).

5. Find \( \int \sin^3 x \cos^4 x \, dx \).

ANSWER: \( \int \sin^3 x \cos^4 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C \).
6. Find \( \int x^3 \sqrt{1 + x^2} \, dx \).

ANSWER: \( \int x^3 \sqrt{1 + x^2} \, dx = \frac{1}{5} (1 + x^2)^{5/2} - \frac{1}{3} (1 + x^2)^{3/2} + C \).

7. Find \( \int \frac{4}{x(x-2)^2} \, dx \).

ANSWER: \( \int \frac{4}{x(x-2)^2} \, dx = \ln |x| - \ln |x-2| - \frac{2}{x-2} + C \).
8. Find \( \int \frac{x^2 - 1}{x^2 + 1} \, dx \).

ANSWER: \( \int \frac{x^2 - 1}{x^2 + 1} \, dx = x - 2 \tan^{-1} x + C. \)

9. Find \( \sum_{k=2}^{\infty} 3^{-2k} \).

ANSWER: \( \sum_{k=2}^{\infty} 3^{-2k} = \frac{3^{-4}}{1 - 3^{-2}}. \)

10. Does the series \( \sum_{k=1}^{\infty} (-1)^k \frac{1}{k} \) converge? If so, does it converge absolutely?

ANSWER: The series converges conditionally.
11. Does \( \sum_{k=3}^{\infty} \frac{k^3 + 5k^2 + 3}{k^6 + 3} \) converge or diverge?

**ANSWER:** The series converges.

12. Find the interval of convergence \( I \) for the power series \( \sum_{k=0}^{\infty} (-1)^k k^{k+1} \).

**ANSWER:** \( I = (-1, 1] \).
13. Find the Taylor series centered at \( a = 0 \) for \( f(x) = \sinh x \).

**ANSWER:** The Taylor series is
\[
\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}.
\]

14. Find \( \lim_{x \to 0} \frac{\sin x - x}{x^3} \) by using Maclaurin series.

**ANSWER:** \( \lim_{x \to 0} \frac{\sin x - x}{x^3} = -\frac{1}{6} \).

15. Graph the polar equation \( r = \cos 2\theta \) and set up an integral that gives the area inside of one leaf.

**ANSWER:** The area is given by \( \int_{\pi/4}^{3\pi/4} \frac{1}{2} \cos^2 2\theta \, d\theta \).
16. Find parametric equations that give a circle centered at \((3, 4)\) with radius 7.

**ANSWER:**

\[
x = 3 + 7 \cos t \\
y = 4 + 7 \sin t.
\]

17. Find the length of the curve \(y = \cosh x\) over \([-\ln 3, \ln 3]\). **Simplify** your answer.

**ANSWER:** \(\frac{8}{3}\).

18. Set up the integral that finds the lateral surface area of the solid generated by rotating the region bounded by \(y = 0, y = x^3, x = 0\) and \(x = 1\) about the \(x\)-axis. You do **not** need to evaluate the integral.

**ANSWER:** \(\int_0^1 2\pi x^3 \sqrt{1 + 9x^4} \, dx\).