I. Introduction

Modeling wildland fire behavior aids decisionmakers who rate fire-danger, plan fire control, and authorize prescribed fires. The forward rate of fire spread, perimeter growth, burning time, intensity, and flame length are not modeled precisely in current practice. The models are useful only for establishing roughly how the fire will behave, calculating relative fire severity, and estimating changes in fire behavior due to changes in burning conditions (for example, windspeed, slopes, and fuel moisture content).

A spreading wildfire includes three interactive processes:

1.1) the **heat sink** characteristics of the unburned fuel;
1.2) the intensity of the flaming **heat source**; and
1.3) the **heat transfer** mechanisms between the flame and the unburned fuel.

![Diagram showing heat source, heat sink, and heat transfer mechanisms](image)

**Figure 1.** Three processes to consider when evaluating a spreading wildfire (Rothermel, 1990).

The rate of fire spread, $R$, through an unburned fuel is expressed as:

$$ R = \frac{l_p}{\rho_{be} \cdot Q_T} $$

where $l_p$ is the propagating heat (or energy flux), W/m²; $\rho_{be}$ is the effective bulk density, a measure of the amount of mass per unit volume that is effectively involved in the absorption of heat during the preignition/ignition phase (kg/m³); and $Q_T$ is the heat of ignition or ignition energy, that is, the net heat absorbed per unit mass (J/kg).

The product $\rho_{be}Q_T$ is the heat absorbed per unit volume of fuel.
I.1) Heat Sink

The effective bulk density differs from the total bulk density of the fuel because it is the fraction of a fuel particle that ignites ahead of the spreading flame front.

$$\rho_{be} = \varepsilon \cdot \rho_b = \rho_b \cdot \exp \left( -\frac{1}{0.22 \cdot \sigma} \right)$$

where $\sigma$ is the ratio of the surface area to the volume of a fuel particle. This relationship is valid for $\sigma > 2.5$ cm$^{-1}$.

The heat required to ignite forest fuels depends on:

a) the specific heat of the fuel and its temperature dependency;

b) $C_v$ and the enthalpy of vaporization of the moisture in the fuel;

c) the enthalpy of the pyrolysis & vaporization products; and

d) $C_p$ of the char residue.

The heat of ignition, $Q_T$, is defined as

$$Q_T = Q_r + m_r \cdot Q_m$$

where $m_r$ = moisture fraction of the fuel, $Q_r$ is the heat of pyrolysis for an assumed combustible vapor ignition temperature of 400°C, and $Q_m$ = the heat necessary to to vaporize this moisture. $Q_r$ is defined as

$$Q_r = \int_{T_{amb}}^{400} \frac{dQ}{dT} dT,$$

and the total energy required for pyrolysis is measured experimentally (Table 1).

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Heat of Pyrolysis (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excelsior (packing shavings)</td>
<td>711</td>
</tr>
<tr>
<td>Ponderosa pine heartwood</td>
<td>721</td>
</tr>
<tr>
<td>Douglas-fir wood (veneer)</td>
<td>781</td>
</tr>
<tr>
<td>Rotten Douglas-fir</td>
<td>522</td>
</tr>
<tr>
<td>Cured white pine needles</td>
<td>557</td>
</tr>
<tr>
<td>Cured ponderosa pine needles</td>
<td>609</td>
</tr>
<tr>
<td>Green ponderosa pine needles</td>
<td>600</td>
</tr>
</tbody>
</table>

$Q_m$ includes the energy required to raise the temperature of the fuel from ambient to the boiling point of water and the energy to boil away moisture:

$$Q_m = C_v \cdot \Delta T + h_g = C_v \cdot (T_b - T_{amb}) + (h_g(T_b) - h_r(T_b)),$$

where $C_v$ is the specific heat of water (4.18 kJ/kg K); $T_b$ is the boiling temperature of water.
(100 °C at STP); and \( h_f \) is the latent heat of vaporization of water at 100 °C (2257 kJ/kg).

**Example 1:** If the moisture content of cured ponderosa pine needles is 5%, calculate the ignition energy.

From Table 1, the heat of pyrolysis is 609 kJ/kg for cured ponderosa pine needles. For an ambient temperature of 32 °C,

\[
Q_T = Q_f + m_r \cdot (C_p \cdot (T_b - T_{amb}) + h_f(T_b))
\]

\[
Q_T = 609 + 0.05 \cdot (4.18 \cdot (100 - 32) + 2257) = 736 \text{ kJ/kg}
\]

How does the ignition energy change for green ponderosa pine needles with a maximum typical moisture content of 30%? \(^1\)

From Table 1, the heat of pyrolysis of green ponderosa pine needles is 600 kJ/kg.

\[
Q_f = 600 + 0.30 \cdot (4.18 \cdot (100 - 32) + 2257) = 1360 \text{ kJ/kg}
\]

Note the nearly 2-fold increase in ignition energy due to the increase in moisture content.

**I.2) Heat Source**

Describing the heat source includes knowing the amount and rate of heat released in the combustion process. The combustion process starts with a rise in temperature of woody material (cellulose and lignin) to a temperature that decomposes these substances into gases, tars and char. Ignition of the pyrolysis decomposition products follows. The total heat of combustion (as measured, for example, in a bomb calorimeter, can be partitioned into flaming (volatile) and glowing (char) components.

**Table 2.** Heat of combustion per kg of original ash-free dry fuel (MJ/kg) (Susott, 1982b).

<table>
<thead>
<tr>
<th>Fuel</th>
<th>No. Samples</th>
<th>Total Heat of Combustion (MJ/kg)</th>
<th>Volatile Heat of Combustion (MJ/kg)</th>
<th>Char Heat of Combustion (MJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foliage</td>
<td>17</td>
<td>22.10</td>
<td>13.12</td>
<td>8.99</td>
</tr>
<tr>
<td>Wood</td>
<td>9</td>
<td>20.29</td>
<td>13.64</td>
<td>6.64</td>
</tr>
<tr>
<td>Stems</td>
<td>4</td>
<td>21.35</td>
<td>13.03</td>
<td>8.32</td>
</tr>
<tr>
<td>Bark</td>
<td>4</td>
<td>22.83</td>
<td>11.41</td>
<td>11.42</td>
</tr>
<tr>
<td>Other</td>
<td>9</td>
<td>20.51</td>
<td>11.40</td>
<td>9.16</td>
</tr>
</tbody>
</table>

\(^1\) The moisture content of fuel at which fire will not spread varies. For litter fuels of ponderosa pine needles, 0.30 is the moisture fraction that will not sustain fire. For other dead fuels, the non-fire moisture content varies between 0.10 and 0.40 (Rothermel, 1972). Logging slash, which is more porous than finer fuels, has a non-fire moisture content between 0.10 and 0.15 (Brown, 1972).
In addition to the heat of combustion, or heat content of the fuel, the amount and rate of fuel consumption is needed to describe the source terms. Two source term models have been used: the fire line intensity model (Byram, 1959) and the reaction intensity model (Rothermel, 1972).

2.a) Fire Line Intensity Source Term

The fire line intensity model describes the intensity of the fire across the line of fire - from the front of the fire to the rear per unit length along the line of fire.

\[ I_B = h \cdot w \cdot R \left( \frac{W}{m} \right) \]

where \( h \) is the heat of combustion or heat content of the fuel (J/kg); \( w \) is the available fuel (kg/m²); and \( R \) = rate of fire spread (m/s). Reasonable correlations have been obtained using this model but \( w \) and \( R \) must be known or measured - which is not practical especially for large or wet fuels where the total fuel load cannot be assumed to burn.

2.b) Reaction Intensity Source Term

The reaction intensity expresses the heat release rate per unit area of the fire.

\[ I_R = -h \cdot \frac{dw}{dt} \left( \frac{W}{m^2} \right) \]

where \( h \) is the heat content of the fuel and \( \frac{dw}{dt} \) is the mass loss rate per unit area; \( I_R \) is a function of fuel size, load, depth, moisture content and heat of combustion. The mass loss rate must be determined experimentally over a wide range of conditions to account for its dependence on fuel size, load, depth, and moisture.

1.3) Heat Transfer

The mechanism of heat transfer from flames to the fuel is hotly debated. The heat needed to sustain steady-state fire spread and the heat of complete combustion can be estimated or measured. But this is not the whole story.

For example, the total energy available from the combustion of foliage is 22,100 kJ/kg from Table 2. The amount of heat needed for ignition of cured ponderosa pine needles with 5% moisture content is 736 kJ/kg, as calculated in Example 1. Hence,

\[ \frac{h_{rel}}{Q_T} = \frac{22,100}{736} = 30 \]

That is, the fuel contains 30 times more energy than is required for ignition. Hence, a heat transfer analysis is required to account for the amount and rate of transfer of heat that causes fire spread.

The heat transfer mechanism debate centers over whether radiation or convection is the dominant method. Conduction through the fuel particle is considered to be much less important because of the porous nature of the fuel arrays (air is a good insulator of conduction) and the slow transfer of heat in woody materials -what is the thermal conductivity and thermal diffusivity of wood ***

For fires backing into the wind, or fires burning on a horizontal surface with more or less vertical flames, radiation from the flames to the unburned fuel is the dominant heat transfer mechanism.
Figure 2. Heat transfer from a fire in a quiescent environment (Rothermel, 1972).

If the air temperature above the unburned fuel is greater than the fuel surface temperature, convective heat transfer is possible in addition to radiation. The result can be faster fire spread than by radiation alone. Situations where this occurs in the wild are wind-driven heading fires (fires burning in the direction of the wind) and fires on slopes. In both these scenarios, there can be considerable convective heating from the flames in addition to radiation. Also, there can be convective cooling as well as radiative cooling from the fire/fuel system.

Figure 3. Heat transfer from the flames of a wind-driven fire (Rothermel, 1972).
II. The Rothermel semi-empirical model for wildfire spread

The Rothermel model starts with an expression that can be evaluated with experimental data. All heat fluxes, whether heating or cooling, are evaluated from experiments wherein the rate of fire spread is measured and the ignition energy is determined from separate, pre-fire experiments.

\[ I_P = R \cdot Q_T \left( \frac{W}{m^2} \right) \]

???****what happened to \( \rho_{be} \)?

Extensive experiments were done with different fuel arrangements and environmental conditions, including wind-driven fires and fires up slopes. The reaction intensity was included:

\[ I_R = w \cdot h \cdot \Gamma' \cdot \eta_m \cdot \eta_s \left( \frac{W}{m^2} \right) \]

where \( w \) is the fuel load (kg/m²); \( h \) is the heat content or heat of combustion (J/kg); \( \Gamma' \) is the reaction velocity (sec⁻¹); \( \eta_m \) is the moisture damping coefficient; and \( \eta_s \) is the mineral damping coefficient. Also, \( \beta \) is the packing ratio, defined by the fraction of the fuel array volume occupied by fuel.

The propagating flux \( I_P \) is related to the reaction intensity \( I_R \) by evaluating the propagating flux ratio, \( \xi \):
\[ \xi = \frac{l_p}{l_R} \]

The completed fire spread model is of the form:

\[ R = \frac{l_R \cdot \xi \cdot (1 + \phi_W + \phi_S)}{\rho_b \cdot \varepsilon \cdot \dot{Q}_T} \left( \frac{m}{\text{sec}} \right) \]

Where \( \phi_W \) and \( \phi_S \) are wind and slope factors, respectively. Also, there is a weighting system used to account for mixed fuel sizes (the model is strongly influenced by the amount of fine fuels in the fuel array since the smaller fuels have the highest area-to-volume ratio). This model works best for uniform fuels when wind and moisture are properly specified.

**** need examples of application, efficacy of model, criticism

III. Summary and Conclusions

IV. Problems

V. References


Table 3. Input data for National Fire-Danger Rating System fuel models (Rothermel, 1972)

<table>
<thead>
<tr>
<th>Fuel Types</th>
<th>Total loading (tons per acre)</th>
<th>1/ft</th>
<th>lb/ft²</th>
<th>1/ft</th>
<th>lb/ft²</th>
<th>1/ft</th>
<th>lb/ft²</th>
<th>1/ft</th>
<th>lb/ft²</th>
<th>Fuel depth (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grass (short)</td>
<td>0.75</td>
<td>3,500</td>
<td>0.034</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>1.0</td>
</tr>
<tr>
<td>Grass (tall)</td>
<td>3.0</td>
<td>1,500</td>
<td>0.138</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>2.5</td>
</tr>
<tr>
<td>Brush (not chaparral)</td>
<td>6.0</td>
<td>2,000</td>
<td>0.046</td>
<td>109</td>
<td>0.023</td>
<td>--</td>
<td>--</td>
<td>1,500</td>
<td>0.092</td>
<td>2.0</td>
</tr>
<tr>
<td>Chaparral</td>
<td>25.0</td>
<td>3,000</td>
<td>0.230</td>
<td>109</td>
<td>0.184</td>
<td>--</td>
<td>0.092</td>
<td>1,500</td>
<td>0.23</td>
<td>6.0</td>
</tr>
<tr>
<td>Timber (grass &amp; understory)</td>
<td>4.0</td>
<td>2,000</td>
<td>0.092</td>
<td>109</td>
<td>0.046</td>
<td>30</td>
<td>0.023</td>
<td>1,500</td>
<td>0.023</td>
<td>1.5</td>
</tr>
<tr>
<td>Timber (litter)</td>
<td>15.0</td>
<td>2,000</td>
<td>0.069</td>
<td>109</td>
<td>0.046</td>
<td>30</td>
<td>0.115</td>
<td>--</td>
<td>--</td>
<td>0.2</td>
</tr>
<tr>
<td>Timber (litter &amp; understory)</td>
<td>30.0</td>
<td>2,500</td>
<td>0.138</td>
<td>109</td>
<td>0.092</td>
<td>30</td>
<td>0.230</td>
<td>1,500</td>
<td>0.092</td>
<td>1.0</td>
</tr>
<tr>
<td>Hardwood (litter)</td>
<td>15.0</td>
<td>1,500</td>
<td>0.134</td>
<td>109</td>
<td>0.019</td>
<td>30</td>
<td>0.007</td>
<td>--</td>
<td>--</td>
<td>0.2</td>
</tr>
<tr>
<td>Logging slash (light)</td>
<td>40.0</td>
<td>1,500</td>
<td>0.069</td>
<td>109</td>
<td>0.207</td>
<td>30</td>
<td>0.253</td>
<td>--</td>
<td>--</td>
<td>1.0</td>
</tr>
<tr>
<td>Logging slash (medium)</td>
<td>120.0</td>
<td>1,500</td>
<td>0.184</td>
<td>109</td>
<td>0.644</td>
<td>30</td>
<td>0.759</td>
<td>--</td>
<td>--</td>
<td>2.3</td>
</tr>
<tr>
<td>Logging slash (heavy)</td>
<td>200.0</td>
<td>1,500</td>
<td>0.322</td>
<td>109</td>
<td>1.058</td>
<td>30</td>
<td>1.288</td>
<td>--</td>
<td>--</td>
<td>3.0</td>
</tr>
</tbody>
</table>
Notes on Table 3:

For all models:

\[ S_f = 0.0555 \]
\[ S_e = 0.010 \]
\[ h = 8,000 \text{ BTU/lb} \]
\[ \rho_p = 32.0 \text{ lb/ft}^3 \]
\[ (M_d)_{\text{dead}} = 0.30 \]

\((M_d)_{\text{living}}\) is determined by the following equation:

\[
(M_x)_{\text{living}} = 2.9 \left( \frac{1 - \alpha}{\alpha} \right) \left[ 1 - \frac{10}{3} (M_f)_{\text{dead}} \right] - 0.226
\]

where \(\alpha\) is the ratio of mass-of-fine-living-fuel to mass-of-total-fine-fuel; fine fuel is taked as fuel \(\leq 1/4\)-inch diameter. \((M_f)_{\text{dead}}\) is the moisture content (a fraction, not percent) of the fine dead fuel.

\[ \tau = \frac{\text{Surface area}}{\text{Volume}} \text{ for a fuel particle} \]
Rothermel's Wildland Fire Spread Model - moisture content

input data

\( w_o := 0.034 \) \hspace{2cm} \text{oovndry fuel loading, lb/ft}^2
\( \delta := 1.0 \) \hspace{2cm} \text{fuel depth, ft}
\( \sigma := 3500 \) \hspace{2cm} \text{fuel particle surface area to volume ratio, 1/ft}
\( h := 8000 \) \hspace{2cm} \text{fuel particle low heat content, BTU/lb}
\( \rho_p := 32 \) \hspace{2cm} \text{oovndry particle density, lb/ft}^3
\( i := 0..30 \) \hspace{2cm} \text{fuel particle moisture content, lb moisture/lb oovndry wood}

\( S_T := 0.0555 \) \hspace{2cm} \text{fuel particle total mineral content, lb minerals/lb oovndry wood}
\( S_e := 0.010 \) \hspace{2cm} \text{fuel particle effective mineral content, lb silica-free minerals per lb of oovndry wood}
\( U := 0 \) \hspace{2cm} \text{wind velocity at midflame height, ft/min}
\( \phi := 0 \) \hspace{2cm} \text{slope, verticle rise/horizontal distance}
\( M_x := 0.30 \) \hspace{2cm} \text{moisture content of extinction}

\( \rho_b := \frac{w_o}{\delta} \) \hspace{2cm} \text{oovndry bulk density, lb/ft}^3
\( \varepsilon := \exp\left( -\frac{138}{\sigma} \right) \) \hspace{2cm} \text{effective heating number}
\( Q_{i\delta_i} := 250 + 1116 \cdot M_{f_i} \) \hspace{2cm} \text{heat of preignition, BTU/lb}
\( \beta := \frac{\rho_b}{\rho_p} \) \hspace{2cm} \text{packing ratio}
\( \beta_{op} := 3.348 \sigma^{0.8189} \) \hspace{2cm} \text{optimum packing ratio}
\( A := \frac{1}{(4.774 \sigma^{0.1} - 7.27)} \) \hspace{2cm} \text{constant}
\[ \Gamma_{p_{\text{max}}} = \sigma^{1.5} \left( 495 + 0.0594 \cdot \sigma^{1.5} \right)^{-1} \text{ maximum reaction velocity, } 1/\text{ft} \]

\[ I_R = \Gamma_{p_{\text{max}}} \left( \frac{\beta}{\beta_{\text{op}}} \right)^A \cdot \exp \left[ A \cdot \left( 1 - \frac{\beta}{\beta_{\text{op}}} \right) \right] \text{ reaction intensity, BTU/min ft}^2 \]

\[ \xi = (192 + 0.2595 \cdot \sigma)^{-1} \cdot \exp \left[ (0.792 + 0.681 \cdot \sigma^{0.5}) \cdot (\beta + 0.1) \right] \text{ propagating flux ratio} \]

\[ C = 7.47 \cdot \exp \left(-0.133 \cdot \sigma^{0.55} \right) \text{ constants} \]

\[ B = 0.02526 \cdot \sigma^{0.54} \]

\[ E = 0.715 \cdot \exp \left(-3.59 \cdot 10^{-4} \cdot \sigma \right) \]

\[ \phi_w = C \cdot U^B \cdot \left( \frac{\beta}{\beta_{\text{op}}} \right)^E \text{ wind coefficient} \]

\[ \phi_s = 5.275 \cdot \beta^{-0.3} \cdot \tan(\phi)^2 \text{ slope factor} \]

\[ R_i = \frac{I_R \cdot \xi \cdot (1 + \phi_w + \phi_s)}{\rho_b \cdot c \cdot Q_{i_1}} \text{ rate of fire spread, ft/min} \]

\[ \eta_{M_i} = 1 - 2.59 \cdot \frac{M_f}{M_x} + 5.11 \cdot \left( \frac{M_f}{M_x} \right)^2 - 3.52 \cdot \left( \frac{M_f}{M_x} \right)^3 \text{ moisture damping coefficient} \]

\[ \eta_s = 0.174 \cdot S c^{-0.19} \text{ mineral damping coefficient} \]

\[ W_n = \frac{w_o}{1 + S \cdot T} \text{ net fuel loading, lb/ft}^2 \]
RESULTS

SHORT GRASS
FLAT
10 MPH WIND

![Graph 1: Fire Spread (ft/min)](image1)

![Graph 2: Heat of Ignition (BTU/h)](image2)
RESULTS

SHORT GRASS
30° SLOPE
NO WIND

![Graphs of Fire Spread and Heat of Ignition vs. Moisture Fraction](image)