FAILRE CRITERIA HANDOUT

Example 1)
An ASTM 30 cast iron has a minimum ultimate strength of 30 kpsi in tension and 100 kpsi in compression. Find the factor of safety if the cast iron is loaded as follows:

$$\sigma_x = 20 \text{ kpsi} \quad \sigma_y = -40 \text{ kpsi}$$

Note that we have a BRITTLE material where

$$S_{ut} = 30 \text{ kpsi} \quad S_{uc} = 10 \text{ kpsi} \quad S_{ut} \neq S_{uc}$$

Which will lead us to use the Mod. II Mohr theory.

Through Morh's circle analysis:

$$\sigma_1 = 20 \text{ kpsi} \quad \sigma_3 = -40 \text{ kpsi}$$

$$\frac{n \cdot \sigma_1}{S_{ut}} = 1 - \left( \frac{n \cdot \sigma_3 + S_{ut}}{-S_{uc} + S_{ut}} \right)^2$$

From this expression \( n = 1.333 \):

Example 2)
A T6 319 cast aluminum fixture is loaded as follows:

$$\sigma_x = 72 \text{ kpsi} \quad \sigma_y = 12 \text{ kpsi}$$

Note that we have a BRITTLE material where

$$S_{ut} = S_{uc} \quad S_{ut} = 36$$

Which will lead us to use the Maximum-Normal-Stress Hypothesis theory.

Through Morh's circle analysis:

$$\sigma_1 = 72 \text{ kpsi} \quad \sigma_3 = 12 \text{ kpsi}$$

$$|\sigma_1| \geq |\sigma_3| \quad \text{Therefore} \quad n = \frac{S_{ut}}{\sigma_1}$$

From this expression: \( n = 0.5 \)

Note that an \( n < 1 \) signifies failure of the component.
Example 3)
This problem was taken from Shigley's Mechanical Engineering Design 5th Ed.

A hot-rolled bar of ductile material has a minimum yield strength in tension and compression of 44 kpsi. Find the factors of safety for each applicable theory of failure for the following stress states.

d) \( \sigma_x = 11 \) kpsi, \( \sigma_y = 4 \) kpsi, \( \tau_{xy} = 1 \) kpsi cw

Note that we have a DUCTILE material where
\[ S_{ut} := S_{uc} \quad S_{ut} := 36 \]

Which will lead us to use the Distortion-Energy Hypothesis

Through Morh's circle analysis:
\[
\sigma'_1 := 11.1 \text{ ksi} \quad \sigma'_2 := 3.85 \text{ ksi} \quad \sigma'_3 := 0
\]

\[
\sigma' := \sqrt{\frac{\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2}{2}}
\]

\[ \sigma' := 9.79 \text{ ksi} \]

\[ n := \frac{S_{ut}}{\sigma'} \]

From this expression \[ n := 3.67 \]

Example 4)
A ductile component with an ultimate compressive strength of 100 kpsi and an ultimate tensile strength of 50 kpsi is loaded as follows:
\[ \sigma_x := 20 \text{ kpsi} \quad \sigma_y := -30 \text{ kpsi} \quad \tau_{xy} := 15 \]

Note that we have a DUCTILE material where
\[ S_{ut} \neq S_{uc} \]

Which will lead us to use the Coulomb Mohr Hypothesis.

Through Morh's circle analysis:
\[ \sigma_1 := 24.15 \text{ ksi} \quad \sigma_2 := 0 \text{ ksi} \]
Because $\sigma_1 > 0$ and $\sigma_3 < 0$ the material is in quadrant 3

Therefore $n$ can be found from the equation

$$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} := \frac{1}{n}$$

From this expression $n := 1.21$