Power Generation Cycles
Vapor Power Generation
The Rankine Cycle
Power Generation Cycles

• Vapor Power Generation Cycles
  – Working fluid experiences a phase change
  – Example: Steam Power Plant

• Gas Power Generation Cycles
  – Working fluid stays in the vapor or gas phase
  – Example: Gas Turbine Engine

• Internal Combustion Engine (ICE) Cycles
  – The working fluid is air in a closed piston-cylinder
  – Example: Spark ignition ICE
  – Example: Compression ignition ICE
A Simple Vapor Power Plant

In ME 322, we are concerned with subsystem A
The Rankine Cycle - Components

(Heat Exchanger)

Boiler

Turbine

Condenser

Cooling water

Pump

4

3

1

2

$\dot{Q}_{in}$

$\dot{Q}_{out}$

$\dot{W}_p$

$\dot{W}_t$
The Rankine Cycle – A Heat Engine

(Heat Source)

(Heat Sink)
Component Analysis

Turbine
\[ \dot{W}_t = \dot{m}(h_1 - h_2) \]

Condenser
\[ \begin{align*}
\dot{Q}_{out} &= \dot{m}(h_2 - h_3) \\
\dot{Q}_{out} &= \dot{m}_w(h_6 - h_5) \\
\dot{Q}_{out} &\approx \dot{m}_w c_p(T_6 - T_5)
\end{align*} \]

Boiler
\[ \dot{Q}_{in} = \dot{m}(h_1 - h_4) \]

Pump
\[ \dot{W}_p = \dot{m}(h_4 - h_3) \]
Performance Parameters

Thermal Efficiency

$$\eta_{th} = \frac{\text{energy sought}}{\text{energy that costs $$}} = \frac{\dot{W}_t - \dot{W}_p}{\dot{Q}_{in}} = \frac{\dot{m}(h_1 - h_2) - \dot{m}(h_4 - h_3)}{\dot{m}(h_1 - h_4)} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

Heat Rate

$$HR = \frac{\text{energy input to the cycle (Btu)}}{\text{net work output (kW-hr)}} = \frac{\dot{Q}_{in} [\text{Btu/hr}]}{(\dot{W}_t - \dot{W}_p) [\text{kW}]} = \frac{(h_1 - h_4) [\text{Btu/hr}]}{[(h_1 - h_2) - (h_4 - h_3)] [\text{kW}]}$$

Back Work Ratio

$$bwr = \frac{\text{pump work required}}{\text{turbine work delivered}} = \frac{\dot{W}_p}{\dot{W}_t} = \frac{\dot{m}(h_4 - h_3)}{\dot{m}(h_1 - h_2)} = \frac{(h_4 - h_3)}{(h_1 - h_2)}$$
Vapor Power Generation

The Ideal Rankine Cycle
The Ideal Rankine Cycle

- **1-2**: Isentropic expansion through the turbine from a saturated (or superheated) vapor state to the condenser pressure
- **2-3**: Heat transfer from the steam at constant pressure through the condenser to a saturated liquid
- **3-4**: Isentropic process through the pump to the boiler pressure
- **4-1**: Heat transfer to the steam at constant pressure through the boiler to complete the cycle

The ideal Rankine Cycle is *internally reversible*:
- No friction effects

Turbine and pump are reversible and adiabatic:
- Isentropic
The ideal cycle also includes the possibility of superheating the saturated vapor.
Example

Ideal Rankine Cycle with Superheat
Ideal Rankine Cycle with Superheat

**Given:** An ideal Rankine Cycle with water as the working fluid with known properties as shown below.

\[
\begin{align*}
P_1 &= 1600 \text{ psia} \\
T_1 &= 1100^\circ F \\
\dot{m} &= 1.4 \times 10^6 \text{ lbm/hr}
\end{align*}
\]

**Find:**
(a) The net power developed (Btu/hr)
(b) The thermal efficiency
(c) The heat rate
(d) The back work ratio
(e) The mass flow rate of the cooling water
Ideal Rankine Cycle with Superheat

"GIVEN: An ideal Rankine Cycle with superheat as shown"
"This analysis will be performed using array variables in EES. Array variables are printed out in an Array Table. This table is very helpful in cycle analysis because you can form a table of properties for quick reference."

\[
P_1 = 1600 \text{ psia} \\
T_1 = 1100 \degree \text{F} \\
\dot{m} = 1.4 \times 10^6 \text{ lbm/hr}
\]

\[
T_2 = 80 \degree \text{F} \\
T_3 = 60 \degree \text{F} \\
T_4 = 50 \degree \text{F}
\]

s$ = 'steam_iapws'

\[
P[1] = 1600 [\text{psia}] \\
T[1] = 1100 [\text{F}] \\
P[2] = 1 [\text{psia}] \\
x[3] = 0 \\
T[5] = 60 [\text{F}] \\
T[6] = 80 [\text{F}] \\
\dot{m} = 1.4E6 [\text{lbm/hr}]
\]

"FIND: (a) The net power developed (hp) 
(b) The thermal efficiency of the cycle 
(c) The heat rate of the cycle 
(d) The back work ratio of the cycle 
(e) The mass flow rate of the cooling water (lbm/hr)"

Unit Settings: Eng F psia mass deg

\[
\dot{m} = 1.400E+06 [\text{lbm/hr}] \\
s$ = 'steam_iapws'
\]
Ideal Rankine Cycle with Superheat

The net power delivered from the cycle is,

\[ \dot{W}_{net} = \dot{W}_t - \dot{W}_p \]

"SOLUTION:"
"Net power delivery from the cycle"
"The net power developed by the cycle is,"

\[ \dot{W}_{net} = \dot{W}_t - \dot{W}_p \]

The First Law applied to the turbine

\[ \dot{W}_t = \dot{m}(h_1 - h_2) \]

"Applying the First Law to the turbine,"

\[ \dot{W}_{dot\_t} = \dot{m}(h[1] - h[2]) \]

\[ h[1] = \text{enthalpy}(s$, P=P[1], T=T[1]) \]

The enthalpy at the exit of the turbine can be found because the turbine is isentropic,

"At the turbine exit, the pressure and entropy are known (in the Rankine Cycle, the turbine and pump are isentropic!"

\[ h[2] = \text{enthalpy}(s$, P=P[2], s=s[2]) \]

\[ s[2] = s[1] \]

\[ s[1] = \text{entropy}(s$, P=P[1], T=T[1]) \]
Ideal Rankine Cycle with Superheat

The First Law applied to the pump,

\[ \dot{W}_p = \dot{m}(h_4 - h_3) \]

"Applying the First law to the pump,"
\[ W_{\text{dot}}_p \times \text{convert}(\text{hp, Btu/ hr}) = \dot{m} \times (h[4] - h[3]) \]
\[ h[3] = \text{enthalpy}(s$, P=P[3], x=x[3]) \]

"At the pump exit, the pressure and entropy are known,"
\[ h[4] = \text{enthalpy}(s$, P=P[4], s=s[4]) \]
\[ s[3] = \text{entropy}(s$, P=P[3], x=x[3]) \]

Unit Settings: Eng F psia mass deg

\[ \dot{m} = 1.400E+06 \ \text{[lbm/hr]} \quad s$ = \text{'steam_iapws'} \]
\[ W_{\text{net}} = 347636 \ \text{[hp]} \quad \dot{W}_p = 2621 \ \text{[hp]} \]
\[ W_t = 350257 \ \text{[hp]} \]
Ideal Rankine Cycle with Superheat

The thermal efficiency of the cycle is,

\[ \eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} \]

"Thermal efficiency of the cycle"
"The thermal efficiency of the cycle is defined as,"
\[ \text{eta}_\text{th} = \text{W}_\text{dot}_\text{net} \times \text{convert(hp,Btu/hr)}/\text{Q}_\text{dot}_\text{in} \]

The heat transfer rate at the boiler is determined from the First Law,

\[ \dot{Q}_{in} = m(h_1 - h_4) \]

"The heat transfer rate at the boiler is,"
\[ \text{Q}_\text{dot}_\text{in} = \text{m}_\text{dot} \times (h[1] - h[4]) \]
Ideal Rankine Cycle with Superheat

The heat rate and back work ratio are defined as,

\[
HR = \frac{\dot{Q}_{in} \text{[Btu/hr]}}{W_{net} \text{[kW]}} \quad \text{bwr} = \frac{\dot{W}_p}{\dot{W}_t}
\]

"Heat rate of the cycle"
"The heat rate (HR) is defined as,"
\[
HR = \frac{Q_{dot\_in}}{(W_{dot\_net} \text{*convert(hp,kW)})}
\]

"Back work ratio of the cycle,"
"The back work ratio (bwr) is defined as,"
\[
bwr = \frac{W_{dot\_p}}{W_{dot\_t}}
\]

To determine the mass flow rate of the cooling water, draw a system boundary around the condenser that keeps the heat transfer all internal.
Ideal Rankine Cycle with Superheat

For the system identified (red),

\[ \dot{m} h_2 + \dot{m}_w h_5 - \dot{m} h_3 - \dot{m}_w h_6 = 0 \]

\[ \dot{m} (h_2 - h_3) = \dot{m}_w (h_6 - h_5) \]

\[ \dot{m} (h_2 - h_3) = \dot{m}_w c_p (T_6 - T_5) \]

"Mass flow rate of the cooling water"

"To determine the cooling water flow rate, draw a system boundary that keeps the heat transferred between the condensing steam and the cooling water all internal. The First Law applied to this system is,"

\[ \text{m\_dot\_w(h[2] - h[3]) = m\_dot\_w\_cpp_w(T[6] - T[5])} \]

"The heat capacity of the liquid can be estimated as the saturated liquid value at the average temperature"

\[ T_{avg} = (T[5] + T[6])/2 \]

\[ \text{cp\_w = cp(s$, T=T\_avg, x=0)} \]
Ideal Rankine Cycle with Superheat

EES Solution (Key Variables):

\[ \dot{W}_t = 350257 \ [\text{hp}] \]
\[ \dot{m} = 1.400E+06 \ [\text{lbm/hr}] \]
\[ \dot{W}_p = 2621 \ [\text{hp}] \]
\[ \dot{W}_\text{net} = 347636 \ [\text{hp}] \]
\[ \dot{Q}_{\text{in}} = 2.063E+09 \ [\text{Btu/hr}] \]
\[ \eta_{\text{th}} = 0.4288 \]
\[ \text{HR} = 7957 \ [\text{Btu/kW-hr}] \]
\[ \text{bwr} = 0.007483 \]
\[ \dot{m}_w = 5.896E+07 \ [\text{lbm/hr}] \]

- Total turbine power delivery
- Mass flow rate of steam in the cycle
- Pump power required
- Net power delivery from the cycle
- Heat transfer rate at the boiler
- Thermal efficiency of the cycle
- Heat rate of the cycle
- Back work ratio of the cycle
- Mass flow rate of cooling water
Ideal Rankine Cycle with Superheat

Results:

\[ P_1 = 1600 \text{ psia} \]
\[ T_1 = 1100^\circ F \]
\[ \dot{m} = 1.4 \times 10^6 \text{ lbm/hr} \]

\[ \eta_{th} = \frac{(891.2 - 6.7) \text{ MBtu/hr}}{2063 \text{ M/Btu/hr}} = 0.429 \]

\[ \dot{W}_t = 891.2 \text{ MBtu/hr} \]
(350,257 hp)

\[ 2063 \text{ MBtu/hr} = \dot{Q}_{in} \]

\[ HR = \frac{2063 \text{ MBtu/hr}}{(891.2 - 6.7) \text{ MBtu/hr}} \times \frac{3412 \text{ Btu/hr}}{\text{kW}} \]
\[ HR = 7958 \text{ Btu/kW-hr} \]

\[ \dot{W}_p = 6.7 \text{ MBtu/hr} \]

\[ T_5 = 60^\circ F \]
\[ T_6 = 80^\circ F \]

\[ \dot{Q}_{out} = 1178 \text{ MBtu/hr} \]

\[ bwr = \frac{6.7 \text{ MBtu/hr}}{891.2 \text{ MBtu/hr}} = 0.0075 \]