Real Rankine Cycle with Superheat

**Given:** A steam power cycle with known properties and operating parameters as shown below.

\[ P_1 = 1600 \text{ psia} \]
\[ T_1 = 1100^\circ \text{F} \]
\[ \dot{m} = 1.4 \times 10^6 \text{ lbm/hr} \]

**Find:**
(a) The net power developed (hp)
(b) The thermal efficiency
(c) The heat rate
(d) The back work ratio
(e) The mass flow rate of the cooling water
"GIVEN: A steam power cycle with superheat as shown"

s$ = 'steam_iapws'
P[1] = 1600[psia]
T[1] = 1100[F]
P[2] = 1[psia]
x[3] = 0
T_5 = 60[F]
T_6 = 80[F]
m_dot = 1.4E6[lbm/hr]
eta_t = 0.85
eta_p = 0.90

"FIND: (a) The net power developed (hp)
(b) The thermal efficiency of the cycle
(c) The heat rate of the cycle
(d) The back work ratio of the cycle
(e) The mass flow rate of the cooling water (lbm/hr)"
Real World Example

Strategy: Build a property table, then do the thermodynamics.

"Building the property table"
"State 1: P[1], T[1] are known"
\[ h[1] = \text{enthalpy}(s$, P=P[1], T=T[1]) \]
\[ s[1] = \text{entropy}(s$, P=P[1], T=T[1]) \]
\[ x[1] = \text{quality}(s$, P=P[1], T=T[1]) \]

"Isentropic efficiency of the turbine. Allows for determination of h[2]"
\[ \eta_t = (h[1] - h[2])/(h[1] - h_2s) \]

"State 2s: P[2], s_2s = s[1] are known"
\[ h_2s = \text{enthalpy}(s$, P=P[2], s=s_2s) \]
\[ s_2s = s[1] \]

"State 2: P[2], h[2] are known"
\[ T[2] = \text{temperature}(s$, P=P[2], h=h[2]) \]
\[ s[2] = \text{entropy}(s$, P=P[2], h=h[2]) \]
\[ x[2] = \text{quality}(s$, P=P[2], h=h[2]) \]

"State 3: P[3], x[3] are known"
\[ T[3] = \text{temperature}(s$, P=P[3], x=x[3]) \]
\[ h[3] = \text{enthalpy}(s$, P=P[3], x=x[3]) \]
\[ s[3] = \text{entropy}(s$, P=P[3], x=x[3]) \]

Knowing the isentropic efficiency allows for the calculation of the actual exit enthalpy

"Isentropic efficiency of the pump. Allows for determination of h[4]"
\[ \eta_p = (h_4s - h[3])/(h[4] - h[3]) \]

"State 4s: P[4], s_4s = s[3] are known"
\[ h_4s = \text{enthalpy}(s$, P=P[4], s=s_4s) \]
\[ s_4s = s[3] \]

"State 4: P[4], h[4] are known"
\[ T[4] = \text{temperature}(s$, P=P[4], h=h[4]) \]
\[ s[4] = \text{entropy}(s$, P=P[4], h=h[4]) \]
\[ x[4] = \text{quality}(s$, P=P[4], h=h[4]) \]
Now that the properties are all known, the thermodynamics is EESy!

- $P_1 = 1600$ psia
- $T_1 = 1100\,^\circ F$
- $\dot{m} = 1.4 \times 10^6$ lbm/hr

- $\eta_t = 0.85$
- $\eta_p = 0.90$

<table>
<thead>
<tr>
<th></th>
<th>$P_i$ [psia]</th>
<th>$T_i$ [F]</th>
<th>$h_i$ [Btu/lbm]</th>
<th>$s_i$ [Btu/lbm-R]</th>
<th>$x_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1600</td>
<td>1100</td>
<td>1548</td>
<td>1.632</td>
<td>100</td>
</tr>
<tr>
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<td>101.7</td>
<td>1007</td>
<td>1.802</td>
<td>0.9048</td>
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<td>1</td>
<td>101.7</td>
<td>69.72</td>
<td>0.1326</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1600</td>
<td>102.8</td>
<td>75.01</td>
<td>0.1336</td>
<td>-100</td>
</tr>
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</table>

- $P_2 = 1$ psia
- $T_2 = 80\,^\circ F$
- $T_5 = 60\,^\circ F$

- 3
- 4
- 1
- 2

Water $T$ vs. $s$ diagram.
The properties are all identified, the thermodynamics can be done!

The net power developed by the cycle is,

\[ W_{\text{dot.net}} = W_{\text{dot.t}} - W_{\text{dot.p}} \]

Applying the First Law to the turbine,

\[ W_{\text{dot.t}} = m_{\text{dot}} (h[1] - h[2]) \]

Applying the First law to the pump,

\[ W_{\text{dot.p}} = m_{\text{dot}} (h[4] - h[3]) \]

The thermal efficiency of the cycle is defined as,

\[ \eta_{\text{th}} = \frac{W_{\text{dot.net}}}{Q_{\text{dot.in}}} \]

The heat transfer rate at the boiler is,

\[ Q_{\text{dot.in}} = m_{\text{dot}} (h[1] - h[4]) \]

The heat rate (HR) is defined as,

\[ HR = \frac{Q_{\text{dot.in}}}{W_{\text{dot.net}}} \]

The back work ratio (bwr) is defined as,

\[ bwr = \frac{W_{\text{dot.p}}}{W_{\text{dot.t}}} \]

Mass flow rate of the cooling water

To determine the cooling water flow rate, draw a system boundary that keeps the heat transferred between the condensing steam and the cooling water all internal. The First Law applied to this system is,

\[ m_{\text{dot}} (h[2] - h[3]) = m_{\text{dot.w}} \cdot cp_{w} (T_{6} - T_{5}) \]

The heat capacity of the liquid can be estimated as the saturated liquid value at the average temperature

\[ T_{\text{avg}} = \frac{(T_{5} + T_{6})}{2} \]

\[ cp_{w} = cp(s$,T=T_{\text{avg}},x=0) \]
Real World Example

EES Result (Key Variables)

- \( \dot{m} = 1.400E+06 \) [lbm/hr]  
  Mass flow rate of steam in the cycle

- \( \dot{W}_t = 297718 \) [hp]  
  Total turbine power delivery

- \( \dot{W}_p = 2912 \) [hp]  
  Pump power required

- \( \dot{W}_{net} = 294806 \) [hp]  
  Net power delivery from the cycle

- \( \dot{Q}_{in} = 2.062E+09 \) [Btu/hr]  
  Heat transfer rate at the boiler

- \( \eta_{th} = 0.3638 \)  
  Thermal efficiency of the cycle

- \( HR = 9380 \) [Btu/kW-hr]  
  Heat rate of the cycle

- \( bwr = 0.009782 \)  
  Back work ratio of the cycle

- \( \dot{m}_w = 6.565E+07 \) [lbm/hr]  
  Mass flow rate of cooling water
Rankine vs. Real Cycle Comparison

**Rankine Cycle**

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**Steam Power Cycle with Turbine and Pump**

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## Rankine vs. Real Cycle Comparison

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<td>( \dot{m} = 1.400 \times 10^6 ) [lbm/hr]</td>
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<tr>
<td>( \dot{W}_t = 350257 ) [hp]</td>
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<td>( \dot{W}_p = 2621 ) [hp]</td>
<td>( \dot{W}_p = 2912 ) [hp]</td>
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<tr>
<td>( \dot{W}_{\text{net}} = 347636 ) [hp]</td>
<td>( \dot{W}_{\text{net}} = 294806 ) [hp]</td>
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<td>( \dot{Q}_{\text{in}} = 2.063 \times 10^9 ) [Btu/hr]</td>
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</tr>
<tr>
<td>( \eta_{\text{th}} = 0.4288 )</td>
<td>( \eta_{\text{th}} = 0.3638 )</td>
</tr>
<tr>
<td>( \text{HR} = 7957 ) [Btu/kW-hr]</td>
<td>( \text{HR} = 9380 ) [Btu/kW-hr]</td>
</tr>
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<td>( \text{bwr} = 0.007483 )</td>
<td>( \text{bwr} = 0.009782 )</td>
</tr>
<tr>
<td>( \dot{m}_w = 5.896 \times 10^7 ) [lbm/hr]</td>
<td>( \dot{m}_w = 6.565 \times 10^7 ) [lbm/hr]</td>
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*Mass flow rate of steam in the cycle*

*Total turbine power delivery*

*Pump power required*

*Net power delivery from the cycle*

*Heat transfer rate at the boiler*

*Thermal efficiency of the cycle*

*Heat rate of the cycle*

*Back work ratio of the cycle*

*Mass flow rate of cooling water*