Use of Regeneration in Vapor Power Cycles
What is Regeneration?

• Goal of regeneration
  – Reduce the fuel input requirements ($Q_{in}$)
  – Increase the temperature of the feedwater entering the boiler (increases average $T_h$ in the cycle)

• Result of regeneration
  – Increased thermal efficiency

• Energy source for regeneration
  – High pressure steam from the turbines

• Regeneration equipment
  – Feedwater heater (FWH)
  – This is a heat exchanger that utilizes the high pressure steam extracted from the turbine to heat the boiler feedwater
Increased temperature into the boiler due to regenerative heating.
Keeping Track of Mass Flow Splits

Define a mass flow fraction,

\[ y_n = \frac{\dot{m}_n}{\dot{m}_1} = \frac{\text{mass flow rate at any state } n}{\text{mass flow rate entering the HPT}} \]

Determination of the flow fractions requires application of the conservation of mass throughout the cycle and the conservation of energy around the feedwater heater(s).

Note: If a mass flow rate is known or can be calculated, then the flow fraction approach is not necessary!
Regeneration – Closed FWH

There are two types of closed feedwater heaters:

- Closed FWH with Drain Pumped Forward
- Closed FWH with Drain Cascaded Backward
Regeneration – Closed FWH

Example – Closed FWH with Drain Cascaded Backward

\[ y_1 = 1 \]

Diagram of closed feedwater heater system with steam generator, condenser, pump, and trap.
Regeneration – Multiple FWH
Given: A Rankine cycle is operating with one open feedwater heater. Steam enters the high pressure turbine at 1500 psia, 900°F. The steam expands through the high pressure turbine to 100 psia where some of the steam is extracted and diverted to an open feedwater heater. The remaining steam expands through the low pressure turbine to the condenser pressure of 1 psia. Saturated liquid exits the feedwater heater and the condenser.

Find:
(a) the boiler heat transfer per lbm of steam entering the high pressure turbine
(b) the thermal efficiency of the cycle
(c) the heat rate of the cycle
Regeneration Cycle

- $P_1 = 1500$ psia
- $T_1 = 900^\circ F$
- $P_2 = 100$ psia
- $P_3 = 1$ psia
- $P_6 = 100$ psia, $x_6 = 0$
- $P_5 = 100$ psia
- $P_4 = 1$ psia, $x_4 = 0$

Diagram showing the process flow and pressures.
Known Properties

"GIVEN: A regenerative Rankine Cycle as shown"

```
s$ = 'steam_iapws'
P[1] = 1500[psia]
T[1] = 900[F]
P[2] = 100[psia]
P[3] = 1[psia]
P[4] = 1[psia]
x[4] = 0
P[5] = 100[psia]
P[6] = 100[psia]
x[6] = 0
P[7] = 1500[psia]
```

"FIND: (a) heat transfer per lbm of steam entering the HPT  
(b) thermal efficiency of the cycle  
(c) heat rate of the cycle"

The next step is to build the property table
"!Build the property table first!"

"State 1: P[1] and T[1] are known"
- $h[1] = \text{enthalpy}(s,P=P[1],T=T[1])$
- $s[1] = \text{entropy}(s,P=P[1],T=T[1])$
- $x[1] = \text{quality}(s,P=P[1],T=T[1])$

- $h[2] = \text{enthalpy}(s,P=P[2],s=s[2])$
- $x[2] = \text{quality}(s,P=P[2],s=s[2])$
- $T[2] = \text{temperature}(s,P=P[2],s=s[2])$

- $h[3] = \text{enthalpy}(s,P=P[3],s=s[3])$
- $x[3] = \text{quality}(s,P=P[3],s=s[3])$
- $T[3] = \text{temperature}(s,P=P[3],s=s[3])$

"State 4: P[4] and x[4] are known"
- $h[4] = \text{enthalpy}(s,P=P[4],x=x[4])$
- $s[4] = \text{entropy}(s,P=P[4],x=x[4])$
- $T[4] = \text{temperature}(s,P=P[4],x=x[4])$

- $h[5] = \text{enthalpy}(s,P=P[5],s=s[5])$
- $x[5] = \text{quality}(s,P=P[5],s=s[5])$
- $T[5] = \text{temperature}(s,P=P[5],s=s[5])$

"State 6: P[6] and x[6] are known"
- $h[6] = \text{enthalpy}(s,P=P[6],x=x[6])$
- $s[6] = \text{entropy}(s,P=P[6],x=x[6])$
- $T[6] = \text{temperature}(s,P=P[6],x=x[6])$

- $h[7] = \text{enthalpy}(s,P=P[7],s=s[7])$
- $x[7] = \text{quality}(s,P=P[7],s=s[7])$
- $T[7] = \text{temperature}(s,P=P[7],s=s[7])$
The resulting property table ...

<table>
<thead>
<tr>
<th>Sort</th>
<th>$P_i$ [psia]</th>
<th>$T_i$ [F]</th>
<th>$h_i$ [Btu/lbm]</th>
<th>$s_i$ [Btu/lbm·F]</th>
<th>$x_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>1500</td>
<td>900</td>
<td>1429.15</td>
<td>1.5569</td>
<td>100</td>
</tr>
<tr>
<td>[2]</td>
<td>100</td>
<td>327.8</td>
<td>1151.09</td>
<td>1.5569</td>
<td>0.959</td>
</tr>
<tr>
<td>[3]</td>
<td>1</td>
<td>101.7</td>
<td>869.29</td>
<td>1.5569</td>
<td>0.772</td>
</tr>
<tr>
<td>[4]</td>
<td>1</td>
<td>101.7</td>
<td>69.72</td>
<td>0.1326</td>
<td>0</td>
</tr>
<tr>
<td>[5]</td>
<td>100</td>
<td>101.7</td>
<td>70.02</td>
<td>0.1326</td>
<td>-100</td>
</tr>
<tr>
<td>[6]</td>
<td>100</td>
<td>327.8</td>
<td>298.51</td>
<td>0.4743</td>
<td>0</td>
</tr>
<tr>
<td>[7]</td>
<td>1500</td>
<td>329.9</td>
<td>303.10</td>
<td>0.4743</td>
<td>-100</td>
</tr>
</tbody>
</table>

Now, we can proceed with the thermodynamics!
The heat transfer rate at the boiler can be found by applying the First Law,

\[ \dot{Q}_{in} = m_1 (h_1 - h_7) \]

No flow rate information is given. However, we can find the heat transferred per lbm of steam entering the HPT,

\[ q_{in} = \frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_7 \]

"Thermodynamic Analysis of the Cycle"
"The heat transferred per unit mass of steam entering the HPT is,"
\[ q_{in} = h[1] - h[7] \]
Turbine Modeling

The thermal efficiency of the cycle is given by,

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_t - \dot{W}_p}{\dot{Q}_{in}} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1}$$

The turbine power delivered is,

$$\dot{W}_t = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

$$\frac{\dot{W}_t}{\dot{m}_1} = h_1 - \frac{\dot{m}_2}{\dot{m}_1} h_2 - \frac{\dot{m}_3}{\dot{m}_1} h_3$$

$$w_t = \frac{\dot{W}_t}{\dot{m}_1} = h_1 - y_2 h_2 - y_3 h_3$$

The flow fractions need to be determined!
Pump Modeling

There are two pumps in the cycle. Therefore,

\[ \dot{W}_p = \dot{W}_{p1} + \dot{W}_{p2} \]

\[ \dot{W}_p = \dot{m}_4 (h_5 - h_4) + \dot{m}_6 (h_7 - h_6) \]

\[ \frac{\dot{W}_p}{\dot{m}_1} = \frac{\dot{m}_4}{\dot{m}_1} (h_5 - h_4) + \frac{\dot{m}_6}{\dot{m}_1} (h_7 - h_6) \]

\[ w_p = \frac{\dot{W}_p}{\dot{m}_1} = y_4 (h_5 - h_4) + y_6 (h_7 - h_6) \]

Then ... \[ \eta_{th} = \frac{\dot{W}_t / \dot{m}_1 - \dot{W}_p / \dot{m}_1}{\dot{Q}_{in} / \dot{m}_1} = \frac{w_t - w_p}{q_{in}} \]

This is an important step in the analysis. All specific energy transfers need to be based on the same flow rate. The common value is chosen to be the inlet to the high pressure turbine (HPT).
Mass Conservation

The flow fractions must be found.
The easy flow fractions are ...

\[ y_1 = y_6 = y_7 = 1 \]
\[ y_3 = y_4 = y_5 \]

"Determination of flow fractions
Easy ones first!"

\[
\begin{align*}
y[1] &= 1 \\
y[6] &= 1 \\
y[7] &= 1 \\
\end{align*}
\]

Conservation of mass applied to the FWH gives,

\[
\dot{m}_2 + \dot{m}_5 = \dot{m}_6
\]
\[
\frac{\dot{m}_2}{\dot{m}_1} + \frac{\dot{m}_5}{\dot{m}_1} = \frac{\dot{m}_6}{\dot{m}_1}
\]
\[ y_2 + y_5 = y_6 \]
Closing the System

Where is the missing equation? Mass is conserved in the FWH, but so is energy. Therefore, we need to apply the First Law to the FWH,

\[ \dot{m}_2 h_2 + \dot{m}_5 h_5 = \dot{m}_6 h_6 \]

\[ \frac{\dot{m}_2}{\dot{m}_1} h_2 + \frac{\dot{m}_5}{\dot{m}_1} h_5 = \frac{\dot{m}_6}{\dot{m}_1} h_6 \]

\[ y_2 h_2 + y_5 h_5 = y_6 h_6 \]

"The First Law applied to the FWH is,"

The equations can be solved! The result is a new property table with a column for the mass flow fractions.
### Augmented Array

#### The updated property table ...

<table>
<thead>
<tr>
<th>Sort</th>
<th>$P_i$ [psia]</th>
<th>$T_i$ [F]</th>
<th>$h_i$ [Btu/lbm]</th>
<th>$s_i$ [Btu/lbm-F]</th>
<th>$x_i$</th>
<th>$y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>1500</td>
<td>900</td>
<td>1429.15</td>
<td>1.5569</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>[2]</td>
<td>100</td>
<td>327.8</td>
<td>1151.09</td>
<td>1.5569</td>
<td>0.959</td>
<td>0.2114</td>
</tr>
<tr>
<td>[3]</td>
<td>1</td>
<td>101.7</td>
<td>869.29</td>
<td>1.5569</td>
<td>0.772</td>
<td>0.7886</td>
</tr>
<tr>
<td>[4]</td>
<td>1</td>
<td>101.7</td>
<td>69.72</td>
<td>0.1326</td>
<td>0</td>
<td>0.7886</td>
</tr>
<tr>
<td>[5]</td>
<td>100</td>
<td>101.7</td>
<td>70.02</td>
<td>0.1326</td>
<td>-100</td>
<td>0.7886</td>
</tr>
<tr>
<td>[6]</td>
<td>100</td>
<td>327.8</td>
<td>298.51</td>
<td>0.4743</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>[7]</td>
<td>1500</td>
<td>329.9</td>
<td>303.10</td>
<td>0.4743</td>
<td>-100</td>
<td>1</td>
</tr>
</tbody>
</table>

#### From previous analysis,

\[
\eta_{th} = \frac{W_t - W_p}{q_{in}}
\]

\[
w_t = h_1 - y_2h_2 - y_3h_3
\]

\[
w_p = y_4(h_5 - h_4) + y_6(h_7 - h_6)
\]
Cycle Performance Parameters

The heat rate of the cycle is,

\[ HR = \frac{\dot{Q}_{in}}{\dot{W}_{net}} = \frac{\dot{Q}_{in}}{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1} = \frac{q_{in}}{w_t - w_p} \]

"The heat rate of the cycle is,"

\[ HR = \frac{q_{in}}{((w_t - w_p)\text{convert(Btu/lb, kw-hr/lb))}} \]

**EES Solution (Key Variables):**

- \( q_{in} = 1126 \text{ [Btu/lb\_m]} \) \( \text{(a) heat transferred per unit mass of steam entering the HPT} \)
- \( w_t = 500.3 \text{ [Btu/lb\_m]} \) \( \text{Turbine work delivered per lbm entering the HPT} \)
- \( w_p = 4.816 \text{ [Btu/lb\_m]} \) \( \text{Pump work required per lbm entering the HPT} \)
- \( \eta_{th} = 0.44 \) \( \text{(b) Thermal efficiency of the cycle} \)
- \( HR = 7755 \text{ [Btu/kw-hr]} \) \( \text{(c) Heat rate of the cycle} \)

\[ P_1 = 1500 \text{ psia} \]
\[ T_1 = 900^\circ \text{F} \]
\[ P_2 = 100 \text{ psia} \]
\[ P_3 = 1 \text{ psia} \]