Jet Aircraft Propulsion
Aircraft Propulsion

• Thrust produced by increasing the kinetic energy of the air in the opposite direction of flight
• Slight acceleration of a large mass of air
  – Engine driving a propeller
• Large acceleration of a small mass of air
  – Turbojet or turbofan engine
• Combination of both
  – Turboprop engine
Aircraft Gas Turbine Engines

Turboprop
Small commuter planes

Turbofan
Larger passenger airliners

Turbojet
high speeds
The Turbojet

Ideal Turbojet

a-1 Isentropic increase in pressure (diffuser)
1-2 Isentropic compression (compressor)
2-3 Isobaric heat addition (combustion chamber)
3-4 Isentropic expansion (turbine)
4-5 Isentropic decrease in pressure with an increase in fluid velocity (nozzle)
The Turbojet with an Afterburner

The turbine exhaust is already hot. The afterburner reheats this exhaust to a higher temperature which provides a higher nozzle exit velocity.
Turbojet Irreversibilities

Isentropic efficiencies
- Diffuser
- Compressor
- Turbine
- Nozzle

Fluid Friction effects
- Combustion chamber
The Turbojet Model

First Law analysis of the components in the cycle

\[ \dot{Q}_{in} = \dot{m}(h_3 - h_2) \]

Combustion is replaced with a heat transfer

\[ \dot{W}_c = \dot{m}(h_2 - h_1) \]

Air is the working fluid throughout the complete cycle

\[ \dot{W}_t = \dot{m}(h_3 - h_4) \]

\[ \dot{W}_{net} = 0 \quad \therefore \quad \dot{W}_t = \dot{W}_c \]

\[ h_a + \frac{V_a^2}{2g_c} = h_1 + \frac{V_1^2}{2g_c} \quad \rightarrow \quad V_1 \parallel V_a \]

\[ h_1 = h_a + \frac{V_a^2}{2g_c} \]

\[ h_4 + \frac{V_4^2}{2g_c} = h_5 + \frac{V_5^2}{2g_c} \quad \rightarrow \quad V_4 \parallel V_5 \]

\[ V_5 = \sqrt{2g_c \left( h_4 - h_5 \right)} \]
Turbojet Performance

There is no net power output of the turbojet engine. Therefore, the idea of net power and thermal efficiency are not meaningful. In turbojet engines, performance is measured by,

- **Propulsive Force (Thrust)**
  - The force resulting from the velocity at the nozzle exit
- **Propulsive Power**
  - The equivalent power developed by the thrust of the engine
- **Propulsive Efficiency**
  - Relationship between propulsive power and the rate of kinetic energy production
Turbojet Performance

Propulsive Force (Thrust)

\[ F = \left( \frac{mV}{g_c} \right)_{\text{exit}} - \left( \frac{mV}{g_c} \right)_{\text{in}} \]

\[ F = \frac{\dot{m}}{g_c} (V_5 - V_a) \]

In this equation, the velocities are relative to the aircraft (engine). For an aircraft traveling in still air,

\[ V_{\text{aircraft}} = V_{\text{in}} = V_a \]

Propulsive Power

The power developed from the thrust of the engine

\[ \dot{W}_p = F V_{\text{aircraft}} \]

\[ \dot{W}_p = \left[ \frac{\dot{m}}{g_c} (V_5 - V_a) \right] V_a \]
Turbojet Performance – Efficiencies

Overall Efficiency  \[ \eta_{overall} = \eta_{thermal} \eta_{propulsive} \]

Thermal Efficiency  \[ \eta_{thermal} = \frac{\dot{m}_{air} (k e_5 - k e_a)}{\dot{m}_{fuel} (H V_{fuel})} \]

Propulsive Efficiency

\[ \eta_{propulsive} = \frac{\dot{W}_p}{\dot{m}_{air} (k e_5 - k e_a)} = \frac{(\dot{m}_{air} / g_c)(V_5 - V_a)V_a}{2 g_c (V_5^2 - V_a^2)} = \frac{2 (V_5 - V_a)V_a}{(V_5 - V_a)(V_5 + V_a)} \]

\[ \therefore \eta_{propulsive} = \frac{2 V_a}{V_5 + V_a} = \frac{2}{V_5 / V_a + 1} \]
Given: A turbojet engine operating as shown below

Find:

(a) The properties at all the state points in the cycle
(b) The heat transfer rate in the combustion chamber (kW)
(c) The velocity at the nozzle exit (m/s)
(d) The propulsive force (lbf)
(e) The propulsive power developed (kW)
(f) The propulsive efficiency of the engine
Turbojet Example

"GIVEN: Gas Turbine Cycle for Aircraft Propulsion"

a$ = 'air_ha'
P[0] = 26[kPa]
T[0] = 230[K]
Vel_0 = 220[m/s]
P[5] = 26[kPa]
m_dot = 25[kg/s]
PR = 11
eta_c = 0.85
eta_t = 0.90

"FIND: a) Properties at all state points in the cycle
   b) Heat transfer rate at the combustion chamber
   c) Velocity at nozzle exit
   d) Propulsive force developed by the engine
   e) Propulsive power delivered
   f) Propulsive efficiency of the engine"

Note: An array position of [0] is allowed in EES!
Turbojet Example

Strategy: Build the property table first. This will require some thermodynamic analysis. Consider each component in the cycle.

Diffuser

\[ \dot{m} \left( h_0 + \frac{V_0^2}{2g_c} \right) = \dot{m} \left( h_1 + \frac{V_1^2}{2g_c} \right) \]

\[ h_0 + \frac{V_0^2}{2g_c} = h_1 \]

"SOLUTION:"

"Build the property table by considering each component individually. This requires some thermodynamics to be included in the analysis."

"Diffuser"

"State 0: P[0], T[0]"

\[ h[0] = \text{enthalpy}(a$, T=T[0], P=P[0]) \]
\[ s[0] = \text{entropy}(a$, T=T[0], P=P[0]) \]

"State 1: h[1], s[1]=s[0]"

\[ h[0] + (\text{Vel}_0**2/2)\text{convert(m^2/s^2,kJ/kg)} = h[1] \]
\[ P[1] = \text{pressure}(a$, h=h[1], s=s[1]) \]
\[ T[1] = \text{temperature}(a$, h=h[1], s=s[1]) \]
\[ s[1] = s[0] \]
**Turbojet Example**

**Compressor**

\[ \eta_c = \frac{h_1 - h_{2s}}{h_1 - h_2} \]

"Compressor"

eta_c = (h_2s - h[1])/(h[2] - h[1])  
w_c = h[2] - h[1]  
PR = P[2]/P[1]

"State 2s: P[2], s_2s = s[1]"

h_2s = enthalpy(a$,P=P[2],s=s_2s)  
s_2s = s[1]

"State 2: P[2], h[2]"

T[2] = temperature(a$,P=P[2],h=h[2])  
s[2] = entropy(a$,P=P[2],h=h[2])

**Combustion Chamber**

"Combustion Chamber"

"State 3: P[3], T[3]"

h[3] = enthalpy(a$,P=P[3],T=T[3])  
s[3] = entropy(a$,P=P[3],T=T[3])

**Turbine**

\[ \eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \quad w_t = w_c \]

"Turbine"

w_t = w_c

"State 4s: h_4s, s_4s = s[3]"

P[4] = pressure(a$,h=h_4s,s=s_4s)  
s_4s = s[3]

"State 4: P[4], h[4]"

T[4] = temperature(a$,P=P[4],h=h[4])  
s[4] = entropy(a$,T=T[4],P=P[4])
Turbojet Example

Nozzle

“At this point, the property table is complete!”

<table>
<thead>
<tr>
<th>Sort</th>
<th>$P_i$ [kPa]</th>
<th>$T_i$ [K]</th>
<th>$h_i$ [kJ/kg]</th>
<th>$s_i$ [kJ/kg-K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0]</td>
<td>26</td>
<td>230</td>
<td>230.2</td>
<td>6.9908</td>
</tr>
<tr>
<td>[1]</td>
<td>36.84</td>
<td>254.1</td>
<td>254.4</td>
<td>6.9908</td>
</tr>
<tr>
<td>[2]</td>
<td>405.3</td>
<td>544.9</td>
<td>549.8</td>
<td>7.0755</td>
</tr>
<tr>
<td>[3]</td>
<td>405.3</td>
<td>1400</td>
<td>1516.4</td>
<td>8.1298</td>
</tr>
<tr>
<td>[4]</td>
<td>163.3</td>
<td>1151</td>
<td>1220.9</td>
<td>8.1586</td>
</tr>
<tr>
<td>[5]</td>
<td>26</td>
<td>719.5</td>
<td>734.8</td>
<td>8.1586</td>
</tr>
</tbody>
</table>
Turbojet Example

Now, we can continue with the rest of the thermodynamic analysis.

Combustion Heat Transfer Rate

\[ \dot{Q}_{in} = \dot{m}(h_3 - h_2) \]

"Complete the thermodynamic analysis ..."
"Combustion Chamber Heat Transfer Rate"
\[ Q_{dot\_in} = \dot{m}\text{dot}*(h[3] - h[2]) \]

Nozzle – Exit Velocity

\[ \dot{m} \left( h_4 + \frac{V_4^2}{2\gamma_c} \right) = \dot{m} \left( h_5 + \frac{V_5^2}{2\gamma_c} \right) \]

\[ h_4 = h_5 + \frac{V_5^2}{2\gamma_c} \]

"Velocity of the air leaving the nozzle,"
\[ h[4] = h[5] + (\text{Vel}_5^2/2)\text{convert(m^2/s^2,kJ/kg)} \]
Turbojet Example

Now the propulsive parameters can be calculated,

\[
F = \frac{\dot{m}}{g_c} \left(V_5 - V_a\right)
\]

\[
\dot{W}_p = FV_{\text{aer}} = FV_0
\]

\[
\eta_{\text{propulsive}} = \frac{\dot{W}_p}{\dot{m}_{air} \left(k e_5 - k e_0\right)}
\]

"Propulsive Force (Engine Thrust)"

\[
F_{\text{thrust}} = \dot{m}_{\text{dot}} \ast (\text{Vel}_5 - \text{Vel}_0) \ast \text{convert(N,lbf)}
\]

"Propulsive Power"

\[
W_{\text{dot}_p} = (F_{\text{thrust}} \ast \text{convert(lbf,N)} \ast \text{Vel}_0) \ast \text{convert(W,kW)}
\]

"Propulsive Efficiency"

\[
\eta_{p} = \frac{W_{\text{dot}_p}}{KE_{\text{dot}}}
\]

\[
KE_{\text{dot}} = \left(\frac{\dot{m}}{2}\right) \ast (\text{Vel}_5^2 - \text{Vel}_0^2) \ast \text{convert(W,kW)}
\]
Turbojet Example

Solution (Key Variables):

\[ Q_{in} = 24164 \text{ [kW]} \]  
(b) Heat transfer rate at the combustion chamber

\[ V_{e_0} = 220 \text{ [m/s]} \]  
Velocity into the engine

\[ V_{e_5} = 986 \text{ [m/s]} \]  
(c) Velocity leaving the engine

\[ F_{thrust} = 4305 \text{ [lbf]} \]  
(d) Propulsive force developed (thrust)

\[ \dot{W}_p = 4213 \text{ [kW]} \]  
(e) Propulsive power developed by the engine

\[ \dot{KE} = 11548 \text{ [kW]} \]  
Net kinetic energy rate leaving the engine

\[ \eta_p = 0.3648 \]  
(f) Propulsive efficiency
How is the energy input to this engine distributed?

\[ \dot{Q}_{in} = 24,164 \text{ kW} \]

\[ P_5 = 26 \text{ kPa} \]
\[ T_5 = 719.5 \text{ K} \]
\[ V_5 = 986 \text{ m/s} \]
\[ \dot{m} = 25 \text{ kg/s} \]

excess thermal energy transfer
\[ \dot{Q}_{out} = \dot{m}(h_5 - h_0) = 12,617 \text{ kW} \quad (52.2\%) \]

kinetic energy production rate
\[ \dot{m}(ke_{net}) = \frac{\dot{m}}{2}(V_5^2 - V_0^2) = 11,548 \text{ kW} \quad (47.8\%) \]

\[ \dot{W}_p = 4,213 \text{ kW} \quad (36.5\%) \]

\[ \dot{m}(ke_{excess}) = 7,335 \text{ kW} \quad (63.5\%) \]