Understanding Error Propagation and the Root Sum Square Method

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'Heating Value' is a very important fuel property – one that cannot be directly measured. One common way to experimentally find this property is with a bomb calorimeter. When doing this experiment, temperature is measured before and after the contained explosion, and knowing information about the mass of fuel, energy of the fusing wire, and the calorimeter constant, the heating value can be calculated with the following formula:

$$HV = \frac{(\Delta T)\cdot W - E_{fuse}}{m_{fuel}}$$

HV – Heating value of the fuel [MJ/kg]
ΔT – Temperature rise [Kelvin]
W – Calorimeter constant [MJ/Kelvin]
E_{fuse} – Energy input to ignite the mixture [MJ]
m_{fuel} – Mass of fuel used [kg]

The nominal value and standard deviation for each variable is in the table below

Parameter	Nominal value	Std. Deviation (ε)
W	10.0 [MJ/K]	0.5 [MJ/K]
E_{fuse}	0.5 [MJ]	0.1[MJ]
m_{fuel}	0.5 [kg]	.05 [kg]
ΔT	1.5 [K]	0.1 [K]

1) Find the nominal value of the Heating Value for this fuel

$29 \frac{MJ}{kg}$

2) Assuming 'worst case,' what is the upper limit of the Heating Value if all errors are two Std. Deviations away (95% confidence)?

1 std $\frac{(1.6)(10.5) - 0.4}{0.45} = 36.4$ error ~ 7.4

2 std $\frac{(1.7)(11) - 0.3}{0.4} = 46$ error ~ 17 → not linear

3) Assuming 'worst case,' what is the lower limit of the Heating Value if all errors are two Std. Deviations away (95% confidence)?

1 std $\frac{(1.4)(9.5) - 0.6}{0.55} = 23.1$ error ~ 5.9

2 std $\frac{(1.3)(9.0) - 0.7}{0.6} = 18.3$ error ~ 10.7

4) Does this range represent the Heating Value to a 95% confidence interval? Why or why not? If not, would this range represent a higher or lower confidence interval than 95%?

1 std $23.1 < HV < 36.4$
2 std $18.3 < HV < 46$
worst case

1 std $25.2 < HV < 32.8$ to 67% confidence
2 std $21.3 < HV < 36.7$ to 95% confidence
3 std $17.51 < HV < 40.49$ to 99% confidence

Notes for ME 430 ~~calc~~ example from activity

$$HV = \frac{(\Delta T)(W) - E_f}{m_f} \qquad \frac{\Delta T\, W}{m_f} - \frac{E_{fuse}}{m_{fuel}}$$

$$\frac{\partial HV}{\partial \Delta T} \quad \frac{W}{m_f}$$

$$HV = \frac{(1.5)(10) - 0.5}{0.5} = 29\ MJ/kg$$

$$\frac{\partial HV}{\partial W} = \frac{\Delta T}{m_f}$$

$$\frac{\partial HV}{\partial E_{fuse}} = -\frac{1}{m_f}$$

$$\frac{\partial HV}{\partial m_f} = \frac{-(\Delta T\, W - E)}{m_f^2}$$

$$\epsilon_{HV} = \left[\left(\frac{\partial HV}{\partial \Delta T}\Big|_{x_n = \bar{x}_n} \epsilon_{\Delta T}\right)^2 + (\ ) + (\ ) + (\ )\right]^{1/2}$$

$$\left(\frac{10.0\ MJ/k}{0.5\ kg}(0.1\ k)\right)^2 = 4\left(\frac{MJ}{kg}\right)^2 \qquad \frac{4}{14.7} = 27\%$$

$$\left(\left(\frac{1.5\ k}{0.5\ kg}\right)(0.5\ MJ/k)\right)^2 = 2.25\left(\frac{MJ}{kg}\right)^2 \qquad \frac{2.25}{14.7} = 15\%$$

$$\left(-\frac{1}{0.5\ kg}\, 0.1\ MJ\right)^2 \qquad 0.04\left(\frac{MJ}{kg}\right)^2 \qquad \frac{0.04}{14.7} = 0.3\%$$

$$\left[\left(-\frac{(1.5(10) - 0.5)}{(0.5)^2}\right) 0.05\ kg\right]^2 = 8.41\left(\frac{MJ}{kg}\right)^2 = 57\%$$

$$\epsilon_{HV} = (4 + 2.25 + 0.04 + 8.41)^{1/2} = 3.83 \qquad \epsilon_{HV}^2 = 14.7$$

$\frac{3.83}{29}$ = 13% error @ 67% confidence

$\frac{7.66}{29}$ 26% error @ 95% confidence