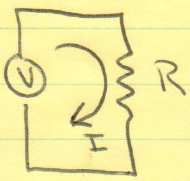
RSS Part 2

Example from last lecture



Want I

Measure V & R

$V = 1.0V \quad \mathcal{E}_V = 0.1V$

$R = 1.0\Omega \quad \mathcal{E}_R = 0.1\Omega$

} 1 std deviation 67% confident

worst case analysis

$$I_{highest} = \frac{V_{high}}{R_{low}} \qquad I_{lowest} = \frac{V_{low}}{R_{High}}$$

$$0.818\,A < I < 1.22\,A$$

* Break for RSS Activity *

Q4) Interval does <u>not</u> represent 67%.

- Does not give confidence interval
- Very conservative tolerance range
- Represents each variable @ extreme simultaneously
- Actually closer to 90^+% confident in this case

RSS Method

given: $y = f(x_1, x_2, x_3, \ldots\ldots x_n)$

y - performance index or dependant variable

x_i - component measurement or independent variable

One std. deviation on each $x_i = \mathcal{E}_{x_i}$

RSS Part 2

$$\varepsilon_y = \left[\left(\frac{\partial f}{\partial x_1}\bigg|_{x_n=\bar{x}_n} \varepsilon_{x_1}\right)^2 + \left(\frac{\partial f}{\partial x_2}\bigg|_{x_n=\bar{x}_n} \varepsilon_{x_2}\right)^2 + \ldots \left(\frac{\partial f}{\partial x_n}\bigg|_{x_n=\bar{x}_n} \varepsilon_{x_n}\right)^2\right]^{1/2}$$

ε_y = One std. deviation tolerance on y

$\frac{\partial f}{\partial x_1} \rightarrow$ equation $\rightarrow$ fill w/ numbers of $\bar{x}_n \rightarrow$ #

ε_{x_1} = # · represents tollerance on measurement 67% of the time

$y - \varepsilon_y <$ true value of $y < y + \varepsilon_y$
$y - 2\varepsilon_y <$ - - - - - - - - - -
$y - 3\varepsilon_y <$ . . - - - - - - - - -

$\boxed{\text{to a 67\% confidence interval; to a 95\% confidence interval; to a 99\% confidence interval}}$

Tolerance on y is __meaningless__ w/o given ~~[illegible]~~ Confidence Interval

ex from before

$y = I \quad x_1 = V \quad x_2 = R \qquad f = \frac{V}{R}$

$\varepsilon_V \quad \frac{\partial f}{\partial V} = \frac{1}{R} \quad @ \bar{x}_1 = \frac{1}{1}$

$\varepsilon_R \quad \frac{\partial f}{\partial R} = \frac{-V}{R^2} \quad @ \bar{x}_2 = -\frac{1}{1^2}$

$$\varepsilon_y = \left[\underbrace{(1 \cdot 0.1)^2}_{\varepsilon_V} + \underbrace{(1 + 0.1)^2}_{\varepsilon_R}\right]^{1/2} = 0.14$$

$0.86 < I < 1.14$ to 67% confidence

$0.72 < I < 1.28$ to 95% confidence

$0.818 < I < 1.22$ ~ 90% confidence – "worst case"

% uncertainty $= \frac{\varepsilon_y}{y}$ @ 67% confidence

$\frac{2\varepsilon_y}{y}$ @ 95% confidence

$\frac{0.14 \text{ amp}}{1 \text{ amp}} \times 100 = 14\%$ @ 67%

$\frac{0.28}{1} \times 100 = 28\%$ @ 95%

RSS Method Part 2

Effects of nominal value

Let $\bar{R} = 2.0\,\Omega \rightarrow \bar{I} = 0.5$ Amps

$$\frac{\partial f}{\partial V} = \frac{1}{2} \qquad \frac{\partial f}{\partial R} = -\frac{1}{4}$$

$\varepsilon_y = 0.14$ previous

$$\varepsilon_y = \Big[\underbrace{\left(\tfrac{1}{2} \cdot 0.1\right)^2}_{\varepsilon_V} + \underbrace{\left(-\tfrac{1}{4} \cdot 0.1\right)^2}_{\varepsilon_R} \Big]^{1/2} = 0.056$$

$$\boxed{0.44 < I < 0.56 \qquad 67\%\text{ confidence}}$$

Ways to improve error

1. Lower $\varepsilon_{x_n} \rightarrow$ \$\$ equipment
2. Lower ε_V — greater impact than ε_R in above example
3. other ways too
4. Some errors given as function of nominal value — above not true in that case

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Design w/ RSS

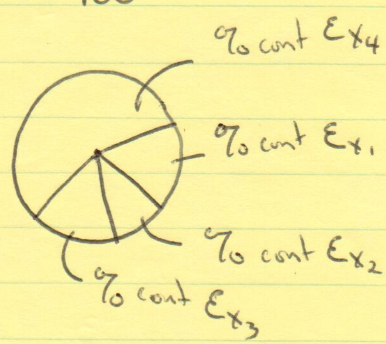
goal on ε_Y, find ε_{x_i}'s to meet

$$\varepsilon_Y^2 = \left(\frac{\partial f}{\partial x_1}\Big|_{x=\bar{x}} \varepsilon_{x_1}\right)^2 + \left(\frac{\partial f}{\partial x_2}\Big|_{x=\bar{x}} \varepsilon_{x_2}\right)^2 + \ldots\ldots \left(\quad\right)^2$$

% contribution to total error

$$\frac{\left(\frac{\partial f}{\partial x_i}\Big|_{x=\bar{x}} \varepsilon_{x_i}\right)^2}{\varepsilon_Y^2} * 100$$

Plot on pie chart



RSS Method Part 2

Safe assumptions on error

ε is 5% of measured value – regular equipment
ε is 2% " " – expensive models
ε is < 2% " " – unconventional – very expensive

HW Hints   HW on overhead

equation to get density is from Archimedes Principle

- story about crown
- find in Fluids or Physics books

- for probs 4 + 5
  - not just few sentences. give details of procedure, and show calcs why it will work

  hint: if changing size, must also change mass