Homework 3

MATH 472

1) Please email me your homework as a single pdf file.
2) Show your work clearly. Justify all your answers.

1. Find the pointwise limit of the sequence $\{f_n\}$ where

$$f_n : [-1,1] \to \mathbb{R} \quad \text{with} \quad f_n(x) = \frac{nx}{1+n^2x^2}.$$

2. For each natural number n and each number $x \geq 2$, define

$$f_n(x) = \frac{1}{1+x^n}.$$

Find the function $f : [2,\infty) \to \mathbb{R}$ to which the sequence $\{f_n : [2,\infty] \to \mathbb{R}\}$ converges pointwise. Prove that the convergence is uniform.

3. Consider the series $1 + x + x^2 + x^3 + \dots$. We know that this series converges pointwise to $\frac{1}{1-x}$ for $x \in (-1,1)$. Show that
(a) The convergence is not uniform on $(-1,1)$.
(b) The convergence is uniform on every closed and bounded interval $[-a,a]$ where $a < 1$.

4. For each natural number n and each number x in $[0,1]$, define

$$f_n(x) = nxe^{-nx^2}.$$

Prove that the sequence $\{f_n\}$ converges pointwise on the interval $[0,1]$ to the constant function 0, but that the sequence of integrals $\{\int_0^1 f_n\}$ does not converge to 0. Does this contradict the theorem on uniform convergence of integrable functions? Why, or why not?