

Homework 4

MATH 472

- 1) Please email me your homework as a single pdf file.
- 2) Show your work clearly. Justify all your answers.

1. Find both the radius and interval of convergence for the given series:

(a) $\sum_{k=1}^{\infty} \frac{1}{k} \left(\frac{x}{2}\right)^k$

(b) $\sum_{k=2}^{\infty} \frac{x^k}{(\ln k)^k}$

2. Write the Taylor series representation for $f(x) = \ln x$ at $x = 1$.
3. Using the Weierstrass M-test, show that the series

$$\sum \frac{1}{1 + k^2 x^2}; \quad x \in (1, \infty)$$

converges uniformly.

4. Using the Weierstrass M-test, show that the series

$$\sum_{n=1}^{\infty} \left(\frac{x^n}{n!}\right)^2$$

converges uniformly for $x \in [-a, a]$.

5. Show that the series

$$\sum_{k=1}^{\infty} \cos \frac{x}{k}, \quad x \in \mathbb{R}$$

is divergent. Differentiate this series term by term and show that the resulting series is convergent.

(Remark: Term by term differentiation of a divergent series might result in a convergent series.)