Homework 3

MATH 420

Please email me your homework as a pdf file
All work must be shown clearly. You must justify all your answers.

1. Write the function $f(z) = z^3+z+1$ in the form $f(z) = u(x, y)+iv(x, y)$.

2. Show that
   (a) $\lim_{z\to\infty} \frac{4z^2}{(z-1)^2} = 4$
   (b) $\lim_{z\to 1} \frac{1}{(z-1)^3} = \infty$

3. Find the points where the function $f(z) = (2x^2 + y) + i(y^2 - x)$ is not analytic.

4. Show that
$$f(z) = \begin{cases} \frac{(x^3-y^3)}{(x^2+y^2)} + i\frac{x^3+y^3}{x^2+y^2} , & \text{if } z \neq 0 \\ 0 , & \text{if } z = 0 \end{cases}$$
   satisfies the Cauchy-Riemann equations at $z = 0$, but is not analytic there.

5. Let $u(x, y) = 2x(1 - y)$. Find a function $v(x, y)$ such that $f(z) = u + iv$ is analytic. Find $f'(z)$.