

## Homework 4

### MATH 420

Please email me your homework as a pdf file

All working must be shown clearly. You must justify all your answers.

1. Evaluate the integral

$$\int_C z \, dz$$

where  $C$  is the straight line from  $z = i$  to  $z = 1$ .

2. Evaluate

$$\int_C (z - 2)\bar{z} \, dz$$

where  $C$  is the arc of the parabola  $y^2 = 4x$  with initial point  $(1, -2)$  and terminal point  $(1, 2)$ .

3. Let  $C$  be the straight line from  $z = i$  to  $z = 1$ . Show that

$$\left| \int_C \frac{dz}{z^4} \right| \leq 4\sqrt{2}$$

without evaluating the integral.

(Observe that of all points on the line segment, the mid-point is the closest to the origin.)

4. Show that

$$\int_C \frac{dz}{(z - a)^n} = \begin{cases} 2\pi i & \text{if } n = 1 \\ 0 & \text{if } n \neq 1 \end{cases}$$

where  $C$  is a simple closed curve around the point  $a$ .

5. Integrate  $\frac{e^{\pi z}}{z^2 + 1}$  in a counter-clockwise sense around the unit circle centered at

(a)  $i$ , (b)  $-i$ , (c)  $1$ , (d)  $-1$

6. Evaluate

$$\int_C \frac{z^3 + 2z}{(z + i)^3} \, dz$$

when  $C$  is a simple closed curve around  $-i$ .