## Homework 5

## MATH 420

Please email me your homework as a pdf file
All working must be shown clearly. You must justify all your answers.

1. Find the region of convergence of the series

$$
\text { (a) } \sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^{2 n-1}}{(2 n-1)!} \quad \text { (b) } \sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^{3} 4^{n}}
$$

2. Find the Maclaurin series of $f(z)=\frac{z}{z^{4}+9}$. Determine the region where the expansion is valid.
Hint: Write $f(z)$ as $\frac{z}{9\left(1+z^{4} / 9\right)}$
3. Find the Taylor series expansion about $z=1$ of $(z-1)^{2} \sin \pi z$.
4. Find a Laurent series expansion of the following about $z=0$ (a) $\frac{1}{z^{2}} e^{z^{3}}$, (b) $\frac{1}{z^{2}(1-z)}$ Indicate the region where the expansion you found is valid.
5. Find a Laurent series expansion of $\frac{1}{z(z-1)^{2}}$ about $z=1$.
