Homework 7

(Will not ge graded)

MATH 420

1. Using the method of "indenting the contour" evaluate

$$\int_{-\infty}^{\infty} \frac{x}{x^4 - 1}\, \mathrm{d}x$$

2. Show that

$$\int_0^{\infty} \frac{\cos x}{a^2 - x^2}\, \mathrm{d}x = \frac{\pi}{2a} \sin a$$

   where a is some positive number.

3. Consider the mapping given by

$$f(z) = 2z^3 + 3z^2 - 12z.$$

   Where would you expect the function to be conformal? Where would you expect it to not be conformal?

4. Check if the given function is harmonic and if so find its harmonic conjugate:

$$H(x, y) = e^x \cos y$$

5. Solve the Dirichlet problem inside a unit circle.
   (This has been done in class. You should verify/work out some of the calculations that were skipped.)
   Next try to imitate the solution for the unit circle to find the solution of the Dirichlet problem inside a circle of radius a.