

Homework 1

MATH 471

- 1) **All work must be shown clearly.** You must justify all your answers.
2) **Problems 1. - 4.** are practice problems on preliminary topics and **will NOT be graded.** However, you are strongly encouraged to try these. **Your score on this homework will be based on problems 5. - 8.**

(Students taking the course through Engineering Outreach may email me your solutions in a pdf file.)

1. (Optional) Let I denote the set of all positive integers, $I = \{1, 2, 3, \dots\}$.
If

$$f(n) = n + 7, \quad (n \in I),$$

$$g(n) = 2n, \quad (n \in I),$$

what is the range of $f \circ g$? What is the range of $g \circ f$?

2. (Optional) (**One-to-one functions**) Which of the following are 1-1 functions? Prove or disprove your answer.

(a) $f(x) = e^{x^2}$, $(-\infty < x < \infty)$

(b) $f : \{x : x \neq 1\} \rightarrow \mathbb{R}$, $f(x) = \frac{x^2-1}{x-1}$

3. (Optional) Find a bijection between the intervals $[0, 1]$ and $[a, b]$.

4. (Optional) (**Onto functions**) Let $A = B = (-\infty, \infty)$. Which of the following functions map A **onto** B ? Justify.

(a) $f(x) = 3$

(b) $f(x) = x$

(c) $f(x) = e^x$

5. Find the least upper bound (l.u.b.) and greatest lower bound (g.l.b.) of $\{\pi + 1, \pi + \frac{1}{2}, \pi + \frac{1}{3}, \dots\}$.

6. Let $S = \{x \in \mathbb{R} : x^2 + x < 3\}$. Find the supremum and infimum of the set S .

7. Find $N \in \mathbb{N}$ such that $\frac{1}{\sqrt{n+1}} < 0.02$ for all $n \geq N$.

8. For each of the following sequences determine whether the sequence converges or diverges. Find the limit for convergent sequences. In each case, prove your conclusion.

(a) $\{a_n\} = \{\frac{1}{2n-3}\}$

(b) $\{a_n\} = \{\frac{n}{n^2-2}\}$