Homework 2

MATH 471

All work must be shown clearly. You must justify all your answers.
(Students taking the course through Engineering Outreach may email me your
solutions in a pdf file.)

1. For each of the following sequences determine whether the sequence con-
   verges or diverges. Find the limit if the sequence converges. In each case,
   prove your conclusion.

   (a) $\{a_n\}_{n=1}^{\infty}$ where
   $$a_n = \begin{cases} 0 & \text{if } n \text{ is odd} \\ \frac{1}{n} & \text{if } n \text{ is even.} \end{cases}$$

   (b) $\{a_n\}_{n=1}^{\infty}$ where
   $$a_n = \begin{cases} 1 & \text{if } n \text{ is odd} \\ \frac{1}{n} & \text{if } n \text{ is even.} \end{cases}$$

2. Determine whether the sequence $\{a_n\}_{n=1}^{\infty}$ with $a_n = \frac{n}{2^n}$ is convergent by
   deciding on monotonicity and boundedness.

3. Let
   $$s_n = \frac{1 + 2 + \cdots + n}{n^2}, \quad n \in \mathbb{N}.$$

   Show that $\{s_n\}_{n=1}^{\infty}$ is monotone and bounded. Find the limit of the se-
   quence $\{s_n\}$.

4. Use the Squeeze/Sandwich Theorem to show that the sequence $\{a_n\}_{n=1}^{\infty}$
   where $a_n = \sqrt{n+1} - \sqrt{n}$ converges. Find the limit.
   Remark: In class we have shown that this sequence converges by showing
   that it is monotonically decreasing and bounded below.