

Homework 3

MATH 471

All work must be shown clearly. You must justify all your answers.
(Students taking the course through Engineering Outreach may email me your solutions in a pdf file.)

1. Define

$$f(x) = \begin{cases} 11 & \text{if } 0 \leq x \leq 1 \\ x & \text{if } 1 < x \leq 2 \end{cases}$$

At what points is the function $f : [0, 2] \rightarrow \mathbb{R}$ continuous? Justify your answer.

2. Suppose that the function $f : [0, 1] \rightarrow \mathbb{R}$ is continuous and that

$$f(x) \geq 2 \quad \text{if } 0 \leq x < 1.$$

Show that $f(1) \geq 2$.

3. Let $f : [0, 1) \rightarrow \mathbb{R}$ be defined by $f(x) = x$. Does f attain a maximum in $[0, 1)$? Does this contradict the Extreme Value Theorem for continuous functions? Why or why not?
4. Define $f(x) = mx + b$ for all x and for fixed constants m and b . Prove that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ is uniformly continuous.
5. Define $f(x) = x^3$ for all x . Prove that $f : \mathbb{R} \rightarrow \mathbb{R}$ is not uniformly continuous.