Homework 4

MATH 471

All work must be shown clearly. You must justify all your answers.
(Students taking the course through Engineering Outreach may email me your solutions in a pdf file.)

1. Use the definition of derivative to compute the derivative of the following functions at $x = 1$:

   (a) $f(x) = \sqrt{x+1}$ for $x > 0$.

   (b) $f(x) = \frac{1}{1+x^2}$ for all x.

2. For what values of a and b is the function $g : \mathbb{R} \to \mathbb{R}$ as defined below differentiable at $x = 1$? Justify your answer.

$$g(x) = \begin{cases} 3x^2 & \text{if } x \leq 1 \\ a + bx & \text{if } x > 1 \end{cases}$$

3. Suppose that the function $f : \mathbb{R} \to \mathbb{R}$ is differentiable and monotonically increasing. Show that $f'(x) \geq 0$ for all x.
   (Note: A function is said to be differentiable if it is differentiable at every $x \in \mathbb{R}$.)

4. Suppose that the function $h : \mathbb{R} \to \mathbb{R}$ is strictly monotone, differentiable, and $h'(x) > 0$ for all x. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable and define $g(x) = f(h^{-1}(x))$ for all x. Find $g'(x)$.