Homework 5

MATH 471

All work must be shown clearly. You must justify all your answers. (Students taking the course through Engineering Outreach may email me your solutions in a pdf file.)

1. Give an example of a function f that is continuous at $x = a$, not differentiable at $x = a$, but yet f attains a relative extremum at $x = a$.

2. Applications of the Mean Value Theorem:

   (a) Let the function f be continuous on $[a, b]$, differentiable on (a, b), and $f'(x) = 0$ on (a, b). Using the Mean Value Theorem show that f must be a constant function on $[a, b]$.

   (b) If functions f and g are continuous on $[a, b]$, differentiable on (a, b), and $f'(x) = g'(x)$ on (a, b), then there exists a real number k such that $f(x) = g(x) + k$ for all $x \in [a, b]$.

   (c) Suppose that f is continuous and differentiable on $[6, 15]$. Suppose $f(6) = -2$ and we know that $f'(x) \leq 10$ for all $x \in [6, 15]$. What is the largest possible value for $f(15)$?

3. Suppose that f is differentiable on some interval D. Prove that if f is Lipschitz then f' is bounded.
   (The converse of this result was proved in the lecture.)