Homework 7

MATH 471

All work must be shown clearly. You must justify all your answers. (Students taking the course through Engineering Outreach may email me your solutions in a pdf file.)

1. If
$$F(x) = \int_0^x \sqrt{t + t^6} \, \mathrm{d}t \quad (x > 0),$$
   find $F'(2)$.

2. Calculate (by using the Fundamental Theorem or one of its corollaries):
$$\frac{\mathrm{d}}{\mathrm{d}x} \left(\int_1^{e^x} \ln t \, \mathrm{d}t \right)$$

3. Suppose that the function $f : \mathbb{R} \to \mathbb{R}$ is differentiable. Define the function $H : \mathbb{R} \to \mathbb{R}$ by
$$H(x) = \int_{-x}^x [f(t) + f(-t)] \, \mathrm{d}t.$$

   Find $H''(x)$.

4. Suppose that the function $f : \mathbb{R} \to \mathbb{R}$ has a continuous second derivative. If
$$g(x) = f(0) + f'(0)x + \int_0^x (x - t)f''(t) \, \mathrm{d}t$$
   then prove that $g'(x) = f'(x)$ for all x.