Math 175 Section 4 Name: ________________________________

Exam 2 Spring 2012

Show all your steps, use correct mathematical notation and simplify your answers to receive credit.

1. Given the sequence $\{a_n\}_{n=1}^{\infty} = \{1, 4, 16, 64, ...\}$
   a) (2 pts) Find the next two terms of the sequence.

   b) (4 pts) Find an explicit formula for the nth term of the sequence in terms of n.

   c) (4 pts) Find a recurrence relation that generates the sequence using the first term and a relation between consecutive terms a_n and a_{n+1}.

2. (8 pts) Determine whether the following sequence converges or diverges by evaluating its limit.

$$\{a_n\} = \left\{ \left(1 + \frac{3}{n}\right)^{2n} \right\}$$

3. (8 pts) Use Simpson's rule with n=4 to approximate the following integral: $\int_0^{\pi} \sin 2x \, dx$

4. (8 pts each) Evaluate the following improper integrals, or determine that they diverge:

a) $\int_{0}^{\infty} xe^{-x}\,dx$

b) $\int_{0}^{25} \frac{dx}{\sqrt{25-x}}$

5. (8 pts each) Evaluate the following integrals using any method.

a) $\int \frac{dx}{\left(4-x^2\right)^{3/2}}$

b) $\int \frac{3}{x^3 - x^2 - 12x} dx$

c) $\int \frac{x^4 + 1}{x^3 + 9x} dx$

d) $\int \frac{2x}{\sqrt{3x+2}} dx$

6. (8 pts) Solve the following initial value problem: $\frac{dy}{dt} = \sqrt{y}\sin t, \quad y(0) = 4$

7. (2 pts each) Multiple choice questions. Circle the correct answer:

a) To evaluate $\int \frac{x^2+1}{x-1}dx$, the first step is to:

A. divide the numerator by the denominator
B. perform a partial fraction decomposition
C. use the substitution u = x – 1.

b) Which of the following is an infinite sequence?
A. {1, 3, 5, 7, … }
B. {2, 4, 6, 8}
C. 1+3+5+7+…

c) The explicit formula $\{a_n\} = \left\{ \frac{\sin\left(\pi n/2\right)}{n} \right\}$ for n=1,2,3,… generates the sequence:

A. {1, 0, -1, 0, 1, …}
B. $\{1, 0\ 1/3, 0, 1/5, \ldots\}$
C. $\{1, 1/2\ 1/3, 1/4, 1/5, \ldots\}$

d) The following ordinary differential equation is separable: $t^2\frac{dy}{dt} = \frac{t+4}{y^2}$

A. True
B. False

Total (out of 100 points):