# Additional Practice Problems/Review Qs for Midterm II

**MATH 471**

1. For two numbers $m_1$ and $m_2$ with $m_1 \neq m_2$, let
$$f(x) = \begin{cases} m_1 x + 4 & \text{if } x \leq 0 \\ m_2 x + 4 & \text{if } x \geq 0. \end{cases}$$
Prove that $f$ is not differentiable at $x = 0$.

2. Let the function $f$ be differentiable at 0. Prove that
$$\lim_{x \to x_0} \frac{x f(x_0) - x_0 f(x)}{x - x_0} = f(x_0) - x_0 f'(x_0).$$
(Hint: Add and subtract $x_0 f(x_0)$ in the numerator of the limit.)

3. Does the Mean Value Theorem apply to the function $g(x) = \sqrt{|x|}$ on $[-1, 1]$? Justify your answer.

4. Prove that if $f$ is differentiable at $c$ and $f$ has an extremum at $c$ then $f'(c) = 0$.
(This has been done in the lecture.)

5. Give an example of a function that attains a relative extremum at $x = a$ but $f'(a) \neq 0$.

6. Suppose that the bounded function $f : [a, b] \to \mathbb{R}$ has the property that for each rational $x$ in the interval $[a, b]$, $f(x) = 0$. Prove that
$$\underline{\int_a^b} f \leq 0 \leq \overline{\int_a^b} f.$$

7. Let
$$f(x) = \begin{cases} x & \text{if the point } x \text{ in } [0,1] \text{ is rational,} \\ 0 & \text{if the point } x \text{ in } [0,1] \text{ is irrational.} \end{cases}$$
Prove that $\underline{\int_0^1} f = 0$ and $\overline{\int_0^1} f \geq 1/2$.

8. Let
$$f(x) = \begin{cases} x & \text{if } 2 \leq x \leq 3 \\ 0 & \text{if } 3 < x \leq 4\ . \end{cases}$$
Prove that $f$ is Riemann integrable.

9. By slightly modifying a proof done in the lecture prove that a monotonically decreasing function is integrable.