# Review Topics for Final Exam

### MATH 430, Fall 2014

The final exam is closed-book, closed-notes, and calculators are not allowed.

- Topics [1]:
  - 1.2 - 1.6: Vector Spaces
    - Vector spaces and subspaces
    - Linear combination; span of a vector space (spanning/generating sets)
    - Linear dependence and linear independence
    - Basis and dimension of vector spaces and subspaces
  - 2.1 - 2.5 : Linear Transformations
    - Linear transformations on general vector spaces; finding the null space and range of a linear transformation; The Dimension Theorem
    - One-to-one & onto linear transformations
    - Finding the matrix of a linear transformation
    - ~~The vector space of linear transformations: addition, scalar multiplication, composition of linear transformations and their matrices~~
    - ~~Invertibility and isomorphism~~
    - ~~The change of coordinate matrix~~
  - 3.x : Solving systems of linear equations
  - 4.1 - 4.3: Determinants and properties
  - 5.1, 5.2: Eigenvalues, eigenvectors, and diagonalizability
  - 6.1: Inner product, norms, orthogonal, and orthonormal sets
  - 6.2: Gram-Schmidt process, orthogonal projections & complements
  - 6.3: Adjoint operator; ~~Least Squares Approximation~~
  - 6.4: Normal and self-adjoint operators
  - 6.5: Unitary matrices and diagonalizing self-adjoint matrices, **application of unitary transformations to conic sections (reducing bilinear forms), unitary operators and their properties**
  - 6.7: **The Singular Value Decomposition**
  - 7.1 - 7.2: **Jordan Canonical Form**

[1]A ~~strikethrough~~ text indicates topics discussed in class but will not appear on the final. A wavy-underlined text indicates topics that will be needed for other topics but there will not be separate questions on these. A **bold-faced** text indicates topics that were covered after the 2nd midterm and should be treated with special emphasis.

- You are expected to be able to prove the following results:

  1. Let $V$ be a vector space, $S_2 \subseteq V$, and $S_1 \subseteq S_2$. If $S_1$ is linearly dependent, then $S_2$ is also linearly dependent. (Week 2)
  2. $B = \{u_1, u_2, \ldots, u_n\}$ is a basis of $V \Leftrightarrow$ every vector in $V$ can be expressed uniquely in terms of elements in $B$. (Week 3)
  3. Let $T : V \to W$ be a linear transformation. Then $N(T)$ and $R(T)$ are subspaces of $V$ and $W$, respectively. (Week 4)
  4. Let $T : V \to W$ be a linear transformation. Then $T$ is one-to-one $\Leftrightarrow$ $N(T) = \{\vec{0}\}$. (Week 4)
  5. Let $\{v_1, v_2, \ldots, v_n\}$ be a basis of $V$. Given n points $\{w_1, w_2, \ldots, w_n\}$ in $W$, there exists a unique linear transformation $T : V \to W$ such that for $i = 1, \ldots, n$, $T(v_i) = w_i$. (Week 5)
  6. An orthogonal set of non-zero vectors is linearly independent. (done in class on 10/24)
  7. The adjoint $T^*$ of a linear operator $T$ is linear.(done in class on 10/31)
  8. Properties of normal operators (done in class on 11/5)
  9. Properties of self-adjoint operators (done in class on 11/7)
  10. $T$ is unitary if and only if $\langle T(x), T(y)\rangle = \langle x, y\rangle$. (done in class on 11/19)
  11. All eigenvalues of an unitary operator have absolute value 1. (done in class on 11/19)

- You are expected to be able to apply **all** theorems/results under the above sections that have been discussed in class.

- Suggestion: Go over the examples solved in class, the problems assigned for the HWs (solutions are posted on the course webpage), and the problems given on the review sheets for the midterms.