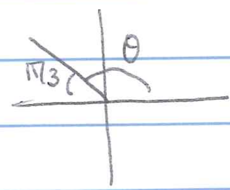
1 (a) $\cos^{-1}\left(-\frac{1}{2}\right)$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = \theta$.

Then $\cos\theta = -\frac{1}{2}$.

$0 \le \theta \le \pi$ $\quad \cos \pi/3 = 1/2$

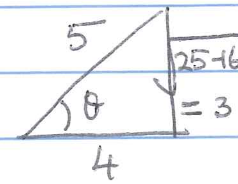
$\Rightarrow \theta = \pi - \pi/3 = \boxed{\frac{2\pi}{3}}$



1 (b) $\sin\left(\cos^{-1}\frac{4}{5}\right)$

Let $\cos^{-1}\frac{4}{5} = \theta$

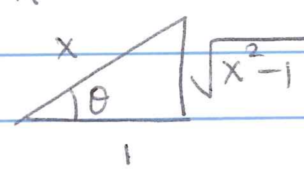
$\Rightarrow \cos\theta = \frac{4}{5}$



$\sin\left(\cos^{-1}\frac{4}{5}\right)$

$= \sin\theta = \boxed{\frac{3}{5}}$

2 $\sin\left(\cos^{-1}\frac{1}{x}\right)$ $\quad$ Let $\cos^{-1}\frac{1}{x} = \theta \Rightarrow \cos\theta = \frac{1}{x}$



$\sin\left(\cos^{-1}\frac{1}{x}\right) = \sin\theta = \frac{\sqrt{x^2-1}}{x}$

3. $y = x\tan^{-1}(2x)$

$\frac{dy}{dx} = 1\cdot\tan^{-1}(2x) + x\,\frac{1}{1+(2x)^2}\,2$ $\quad$ using product & chain rule

$= \tan^{-1}2x + \frac{2x}{1+4x^2}$

4 (a) $\lim_{x\to 0}\frac{1-\cos x}{2x}$ $\left[\frac{0}{0}\right]$

by L'H $= \lim_{x\to 0}\frac{\sin x}{2} = 0$

wrong derivative (-2) or (-1)

$\sin 0 = \phi$ x (-2)

4 (b) $\lim_{x\to\infty} x^{1/x}$ $\left[\infty^0\right]$

$= \lim_{x\to\infty} e^{\ln x^{1/x}}$

$= \lim_{x\to\infty} e^{\frac{1}{x}\ln x}$

$= e^{\lim_{x\to\infty}\frac{\ln x}{x}} = e^0 = 1$

$\lim_{x\to\infty}\frac{\ln x}{x}$ $\left[\frac{\infty}{\infty}\right]$

by L'H $= \lim_{x\to\infty}\frac{1/x}{1}$

$= \lim_{x\to\infty}\frac{1}{x} = 0$

5(a) $\displaystyle\int \frac{x}{\sqrt{x^2-4}}\,dx$ $\qquad u = x^2-4$

$\qquad du = 2x\,dx \Rightarrow x\,dx = \frac{du}{2}$

$$= \frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\,\frac{\sqrt{u}}{1/2} + C = \sqrt{u} + C = \boxed{\sqrt{x^2-4} + C}$$

(b) $\displaystyle\int_1^e \frac{\ln x}{x}\,dx$ $\qquad u = \ln x$

$\qquad du = \frac{1}{x}\,dx$

$$= \int_0^1 u\,du$$

$x = 1 \Rightarrow u = \ln 1 = 0$

$x = e \Rightarrow u = \ln e = 1$

$$= \frac{u^2}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$$

(c) $\displaystyle\int x\cos x\,dx$ $\qquad u = x \qquad dv = \cos x\,dx$

$\qquad du = dx \qquad v = \sin x$

$$= x\sin x - \int \sin x\,dx$$

$$= x\sin x + \cos x + C$$

(d) $\displaystyle\int x^2 e^x\,dx$ $\qquad \boxed{\begin{array}{ll} u = x^2 & dv = e^x dx \\ du = 2x\,dx & v = e^x \end{array}}$

$$= x^2e^x - 2\int xe^x\,dx \qquad \boxed{\begin{array}{ll} u = x & dv = e^x dx \\ du = dx & v = e^x \end{array}}$$

$$= x^2e^x - 2\left[xe^x - \int e^x dx\right]$$

$$= x^2e^x - 2(xe^x - e^x) + C = e^x(x^2 - 2x + 2) + C$$

5 (e) $\int \sin^3 x \cos^4 x \, dx$

$= \int \sin^2 x \cos^4 x \sin x \, dx$ (factor out sin x)

$= \int (1-\cos^2 x) \cos^4 x \sin x \, dx$ use $\sin^2 x = 1 - \cos^2 x$

substitute $\cos x = u$

$-\sin x dx = du$

$= -\int (1-u^2) u^4 \, du$

$= -\int (u^4 - u^6) \, du = \int (u^6 - u^4) du = \frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$