NAME:

MIDTERM II

MATH 175, FALL 2014

11.30 AM - 12.20 PM

Show work clearly for full credit. All steps must be shown.

Notes, books, or calculators cannot be used

1. (16 pts) Consider the sequence $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\}$.

(a) Find the next two terms of the sequence.

$$\frac{1}{32}, \frac{1}{64}$$

(b) Find an explicit formula for the nth term of the sequence.

$$a_n = \frac{1}{2^n}, \quad n = 1, 2, \ldots$$

(c) Find a recurrence relation that generates the sequence.

$$a_n = \frac{a_{n-1}}{2}, \quad a_1 = \frac{1}{2}$$

(d) Does the sequence converge or diverge? If it converges, find the limit.

Converges, geometric sequence $r = \frac{1}{2} < 1$

limit $= 0$

2. (4 pts) Determine whether the following sequence converges or diverges. If it converges, find the limit.

$$\{a_n\} = \left\{2 + \frac{2}{n}\right\}$$

$$\lim_{n \to \infty} \left(2 + \frac{2}{n}\right) = 2 + 0 = 2$$

converges, limit is 2

3. (20 pts) Evaluate the following improper integrals, or show that the integral diverges. Use proper notation.

(a) $\int_2^5 \frac{1}{(x-2)^2}\, dx$

$$\lim_{a \to 2^+} \int_a^5 \frac{1}{(x-2)^2} = \lim_{a \to 2^+} \left[\frac{-1}{x-2} \right]_a^5 = \lim_{a \to 2^+} -\frac{1}{3} + \frac{1}{a-2}$$

$$= \infty$$

diverges

(b) $\int_1^\infty \frac{1}{(x+3)^{3/2}}\, dx$

$$\lim_{b \to \infty} \int_1^b \frac{1}{(x+3)^{3/2}} = \lim_{b \to \infty} \left[\frac{1(-2)}{\sqrt{x+3}} \right]_1^b$$

$$= \lim_{b \to \infty} \left[\frac{-2}{\sqrt{b+3}} + \frac{2}{\sqrt{4}} \right] = 1$$

4. (12 pts each) Evaluate the following integrals. If you make a substitution indicate clearly what it is.

(a)

$$\int \frac{1}{(9-x^2)^{3/2}}dx$$

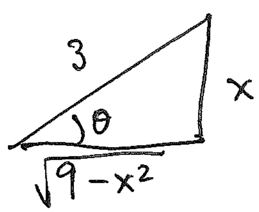
$x = 3\sin\theta \qquad (9-x^2)^{3/2} = (9-9\sin^2\theta)^{3/2}$

$dx = 3\cos\theta\, d\theta \qquad = (9\cos^2\theta)^{3/2} = (3\cos\theta)^3$

$\sin\theta = x/3$

$$\int \frac{3\cos\theta\, d\theta}{27\cos^3\theta} = \frac{1}{9}\int \sec^2\theta\, d\theta = \frac{1}{9}\tan\theta + C$$

$$= \frac{1}{9}\frac{x}{\sqrt{9-x^2}} + C$$



(b)

$$\int \frac{1}{(x-2)(x-1)^2}\, dx$$

$$\frac{1}{(x-2)(x-1)^2} = \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$1 = A(x-1)^2 + B(x-2)(x-1) + C(x-2)$$

$x=1: \quad 1 = -C \quad \Rightarrow \quad \boxed{C=-1}$

$x=2: \quad \boxed{1 = A}$

$x=0: \quad 1 = A + 2B - 2C \quad \Rightarrow \quad 1 = 1 + 2B + 2$

$\Rightarrow \quad B = -1$

$$\int \frac{1}{(x-2)(x-1)^2}dx = \int \frac{1}{x-2} \quad \int \frac{-1}{x-1} - \int \frac{1}{(x-1)^2}$$

$$= \ln|x-2| - \ln|x-1| + \frac{1}{x-1} + C$$

(c)

$$\int \frac{1}{\sqrt{x^2+4}}\,dx$$

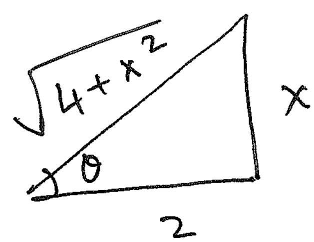
$$x = 2\tan\theta \qquad dx = 2\sec^2\theta\, d\theta$$

$$\sqrt{x^2+4} = \sqrt{4\tan^2\theta + 4} = 2\sec\theta$$

$$\int \frac{2\sec^2\theta\, d\theta}{2\sec\theta} = \int \sec\theta\, d\theta = \ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C$$

$\tan\theta = \frac{x}{2}$



5. (4 pts) Circle the correct answer.
The partial fraction decomposition of the function $\frac{x^2}{x(x+1)^2(x^2+3)}$ is

(A) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{x^2+3}$

(B) $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{Dx+E}{x^2+3}$

(C) $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{x^2+3}$

(D) None of the above

6. (10 pts) Solve the following separable first order differential equation:

$$\frac{dy}{dx} = xy^2$$

Write y explicitly as a function of x.

$$\frac{dy}{dx} = xy^2 \Rightarrow \frac{dy}{y^2} = x\,dx$$

$$\Rightarrow \int \frac{dy}{y^2} = \int x\,dx$$

$$\Rightarrow -\frac{1}{y} = \frac{x^2}{2} + C$$

$$\Rightarrow y = \frac{-1}{\frac{x^2}{2} + C}$$

7. (10 pts) Use the Trapezoidal Rule with $n = 4$ to approximate the integral:

$$\int_0^4 \cos(\pi x)\, dx$$

$n = 4$, $\Delta x = \dfrac{4-0}{4} = 1$, $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$

$f(x) = \cos(\pi x)$

$f(x_0) = f(0) = \cos 0 = 1$

$f(x_1) = f(1) = \cos \pi = -1$

$f(x_2) = f(2) = \cos 2\pi = 1$

$f(x_3) = f(3) = \cos 3\pi = -1$

$f(x_4) = f(4) = \cos 4\pi = 1$

$$\int_0^4 \cos \pi x \, dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$= \frac{1}{2}\left[1 + 2(-1) + 2(1) + 2(-1) + 1\right]$$

$$= \frac{1}{2}(0) = 0.$$