Homework

#14 $$\int \frac{3}{x^3 - x^2 - 12x}\,dx = \int \frac{3\,dx}{x(x^2 - x - 12)} = \int \frac{3\,dx}{x(x-4)(x+3)}$$

$$\frac{3}{x(x-4)(x+3)} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{x+3}$$

$$\Rightarrow \quad 3 = A(x-4)(x+3) + Bx(x+3) + Cx(x-4)$$

Let $x = 0$: $3 = -12A \Rightarrow \boxed{A = -1/4}$

Let $x = 4$: $3 = B(4)(7) \Rightarrow 3 = 28B \Rightarrow \boxed{B = 3/28}$

Let $x = -3$: $3 = C(-3)(-7) \Rightarrow \boxed{C = 1/7}$

$$\int \frac{3}{x^3 - x^2 - 12x}\,dx = \int \frac{(-1/4)}{x}\,dx + \int \frac{(3/28)}{x-4}\,dx + \int \frac{(1/7)\,dx}{x+3}$$

$$= -\frac{1}{4}\ln x + \frac{3}{28}\ln|x-4| + \frac{1}{7}\ln|x+3| + C$$

#20 $$\int \frac{x}{(x-6)(x+2)^2}\,dx$$

$$\frac{x}{(x-6)(x+2)^2} = \frac{A}{x-6} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$x = A(x+2)^2 + B(x-6)(x+2) + C(x-6)$$

Let $x = 6$: $6 = A(64) \Rightarrow \boxed{A = 3/32}$

Let $x = -2$: $-2 = C(-8) \Rightarrow \boxed{C = 1/4}$

Let $x = 0$: $0 = 4A - 12B - 6C$

or, $0 = 4\left(\frac{3}{32}\right) - 12B - 6\left(\frac{1}{4}\right)$

$$\Rightarrow \boxed{B = -3/32}$$

$$\int \frac{x}{(x-6)(x+2)^2}\,dx = \int \frac{(3/32)}{x-6}\,dx + \int \frac{(-3/32)}{x+2}\,dx + \int \frac{(1/4)}{(x+2)^2}\,dx$$

$$= \frac{3}{32}\ln|x-6| - \frac{3}{32}\ln|x+2| - \frac{1}{4}\frac{1}{(x+2)} + C$$