9.4 Divergence & Integral Tests

16 Use the divergence test to determine divergence or state that the test is inconclusive.

$$\sum_{k=1}^{\infty} \frac{k}{k^2+1} \qquad a_k = \frac{k}{k^2+1}, \quad \lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{k}{k^2+1}$$

$$= \lim_{k\to\infty} \frac{1}{k+\frac{1}{k}}$$

$$= 0$$

5 pts.

The divergence test is inconclusive

20 $$\sum_{k=1}^{\infty} \frac{k^3}{k^3+1} \qquad \lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{k^3}{k^3+1}$$

$$= \lim_{k\to\infty} \frac{1}{1+\frac{1}{k^3}} = 1 \neq 0$$

5 pts

By the divergence test, the given series diverges.

26 Use the integral test to deduce convergence or divergence

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k+10}}$$

Let $f(x) = \frac{1}{\sqrt[3]{x+10}}$ is continuous, positive and decreasing on $[1,\infty)$

10 pts

$$\int_1^{\infty} \frac{dx}{(x+10)^{1/3}} = \lim_{b\to\infty} \int_1^b \frac{dx}{(x+10)^{1/3}} = \lim_{b\to\infty} \left[\frac{3}{2}(x+10)^{-1/3+1} \right]_1^b$$

$$= \lim_{b\to\infty} \left[\frac{3}{2}(b+10)^{2/3} - \frac{3}{2}\, 11^{2/3} \right] = \infty$$

. Therefore, the series diverges.

28 Use integral test to determine convergence or divergence.

$$\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$$

Let $f(x) = \dfrac{1}{x(\ln x)^2}$. Then $f(x)$ is continuous, decreasing and positive on $[2, \infty)$

Substitution:

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}}$$

$$\int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx = \lim_{b \to \infty} \int_2^{b} \frac{1}{x(\ln x)^2}\,dx$$

$$= \lim_{b \to \infty} \int_{\ln 2}^{\ln b} \frac{du}{u^2} = \lim_{b \to \infty} \left[-\frac{1}{u}\right]_{\ln 2}^{\ln b}$$

10 pts.

$$= \lim_{b \to \infty} \left[-\frac{1}{\ln b} + \frac{1}{\ln 2}\right] = \frac{1}{\ln 2} < \infty$$

Therefore, the series converges by the integral test.