MATH 471
Review for Final

- The final is comprehensive and will be closed-book, closed notes.
- **Theorems you should be able to state and prove**:
  1. The following part of the Monotone Convergence Theorem: An increasing sequence that is bounded above is convergent. The limit is the supremum of the sequence. (see Lecture 6)
  2. A continuous function on a closed and bounded set is uniformly continuous. (see Lecture 14)
  3. The two Mean Value Theorems of Integration
  4. The two Fundamental Theorems of Calculus
- **Topics, sub-topics, and important results**:
  - Convergence of sequences
    - Supremum (sup or least upper bound or l.u.b.) & infimum (inf or greatest lower bound or g.l.b.) of a set
    - Definition of convergence of a sequence and limit of a convergent sequence
    - Operations on sequences: sum, difference, product, etc. of convergent sequences
    - If a real number c satisfies $|c| < 1$ then $\lim_{n\to\infty} c^n = 0$ and if $|c| > 1$ then $\{c^n\}_{n=1}^{\infty}$ diverges.
    - Monotone sequences: The Monotone Convergence Theorem
    - Subsequences: Every subsequence of a convergent sequence converges to the same limit.
    - Bounded sequences: Every convergent sequence is bounded. Converse is not true. However, every bounded sequence has a convergent subsequence (the Bolzano-Weierstrass Theorem).
    - Sandwich or Squeeze Theorem
  - Continuity
    - Definition of continuity
    - Sum, product, composition of continuous functions is continuous.
    - Uniform continuity: Proving uniform continuity using the definition
      Theorem : If $D \subseteq \mathbb{R}$ is a *closed and bounded set* and $f : D \to \mathbb{R}$ is continuous, then f is uniformly continuous.

    - Extreme and Intermediate Value Theorem and their applications: for example, using the IVT to show that a polynomial of odd degree has at least one root. (see Lecture 12)
    - Limit points of a set and limit of a function at a point. Connection to continuity.
  - Differentiation
    - Definition of differentiability at a point
    - Differentiating sums, products, quotients, inverses and compositions
  - (Riemann) Integration
    - Finding the lower sum, upper sum, lower integral, upper integral of a function
    - Archimedes Riemann Theorem
    - Integrability of monotone and continuous functions
    - Properties of integrals; additivity, linearity, etc.
  - Fundamental Theorems of Calculus
  - Approximation of functions by Taylor polynomials, Lagrange Remainder Theorem

- **Be familiar with all the examples and counter-examples discussed in class. Be familiar with all the problems from Homeworks 1-8. Consider going over the solutions provided for each homework. Redo the practice problems that were provided before midterms I & II.**