

MATH 472
Guidelines for Exam 2

- **Theorems/Lemmas/Propositions you are expected to be able to state and prove:**
 1. $\mathbf{u} \in \mathbb{R}^n$ and $\mathbf{v} \in \mathbb{R}^n$ are orthogonal if and only if the Pythagorean identity holds.
 2. The Cauchy-Schwarz Inequality
 3. The triangle inequality for vectors in \mathbb{R}^n
 4. The intersection of a finite number of open sets is open.
The union of an arbitrary collection of open sets is open.
 5. The following are equivalent:
 - (a) $f : D \rightarrow \mathbb{R}$ is continuous
 - (b) For every open set U in \mathbb{R} , $f^{-1}(U)$ is open in D
(In class, we proved only one direction and that is enough for the test.)

- **Theorems/Lemmas/Propositions you are expected to be able to apply:**
 1. All of the above
 2. All the results discussed for power series
 3. Componentwise Convergence Criteria - A sequence $\{\mathbf{u}_k\}$ in \mathbb{R}^n converges to \mathbf{u} if and only if $\{\mathbf{u}_k\}$ converges componentwise to \mathbf{u} .
 4. The union of a finite number of closed sets is closed.
The intersection of an arbitrary collection of closed sets is closed.
 5. The following are equivalent:
 - (a) $f : D \rightarrow \mathbb{R}$ is continuous at a point \mathbf{x}_0 (the ϵ - δ criterion holds at \mathbf{x}_0)
 - (b) For every convergent sequence $\{\mathbf{x}_k\} \rightarrow \mathbf{x}_0$ in D , $\{f(\mathbf{x}_k)\}$ converges to $f(\mathbf{x}_0)$
 6. Clairaut's Theorem
 7. Directional Derivative Theorem

- **Must be able to clearly state all the definitions:** For example: open ball, interior point, open sets, closed sets, accumulation point, limit and continuity at a point, continuously differentiable function, etc.

- **Be familiar with all the examples discussed in class.**

- **Be familiar with all the problems from Homeworks 4, 5.**

- **Note:** In the lectures, an arrow over a letter is used to indicate that the object is a vector. In print, a boldfaced letter is used to indicate a vector. Be mindful of this while reading questions.