Review Topics & Study Guide for Exam I

MATH 430

The exam is closed-book, closed-notes, and calculators are not allowed.

- Chapter 1: Vector Spaces

  - Vector spaces and subspaces
  - Linear combination; span of a vector space (spanning/generating sets)
  - Linear dependence and linear independence
  - Basis and dimension of vector spaces and subspaces

- Chapter 2: Linear Transformations

  - Linear transformations on general vector spaces; finding the null space and range of a linear transformation; The Dimension Theorem
  - One-to-one & onto linear transformations
  - Finding the matrix of a linear transformation
  - The vector space of linear transformations: addition, scalar multiplication, composition of linear transformations and their matrices
  - Invertibility and isomorphism
  - The change of coordinate matrix

- You are expected to be able to apply **all** theorems/results discussed in class

- You are expected to be able to **prove** the following theorems:

  1. The Cancellation Law of Vector Addition
  2. $B = \{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n\}$ is a basis of $V \Leftrightarrow$ every vector in V can be expressed uniquely in terms of elements in B.
  3. Let $T : V \to W$ be a linear transformation. Then $N(T)$ and $R(T)$ are subspaces of V and W, respectively.
  4. Let $T : V \to W$ be a linear transformation. Then T is one-to-one $\Leftrightarrow$ $N(T) = \{\vec{0}\}$.
  5. If V and W are vector spaces of equal (finite) dimension, and let $T : V \to W$ be linear. Then the following are equivalent.
     (a) T is one-to-one.
     (b) T is onto.
     (c) rank(T) = dim(V).

- See separate file for additional practice problems. Go over the examples solved in class, in the book, and assigned for HWs 1-4.