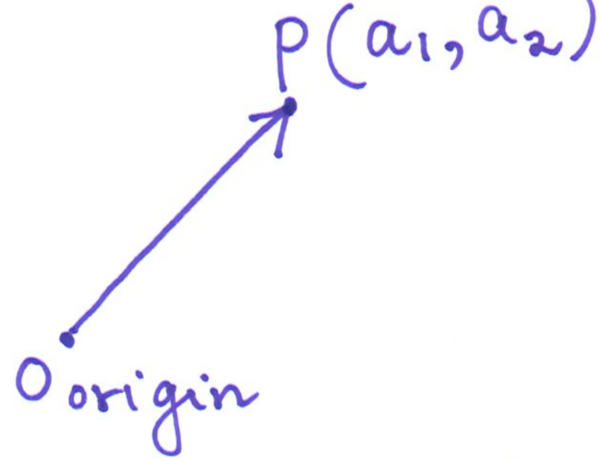
MATH 430
Advanced Linear Algebra

Session 1

Vector Spaces

From Calc III : a vector in a $\underbrace{\text{plane}}_{x-y \text{ plane}}$ is a point p



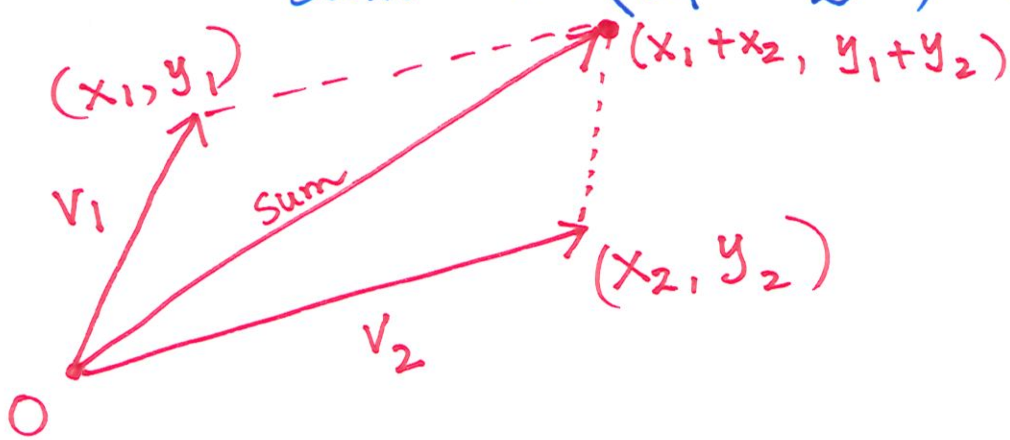
Direction $\overrightarrow{OP}$

Magnitude/length of $\overrightarrow{OP}$:

$$\sqrt{a_1^2 + a_2^2}$$

Addition of vectors (x_1, y_1) and (x_2, y_2)

Sum : $(x_1 + x_2, y_1 + y_2)$ is another vector



$\mathbb{R}$ - real numbers , $\mathbb{C}$ - complex numbers

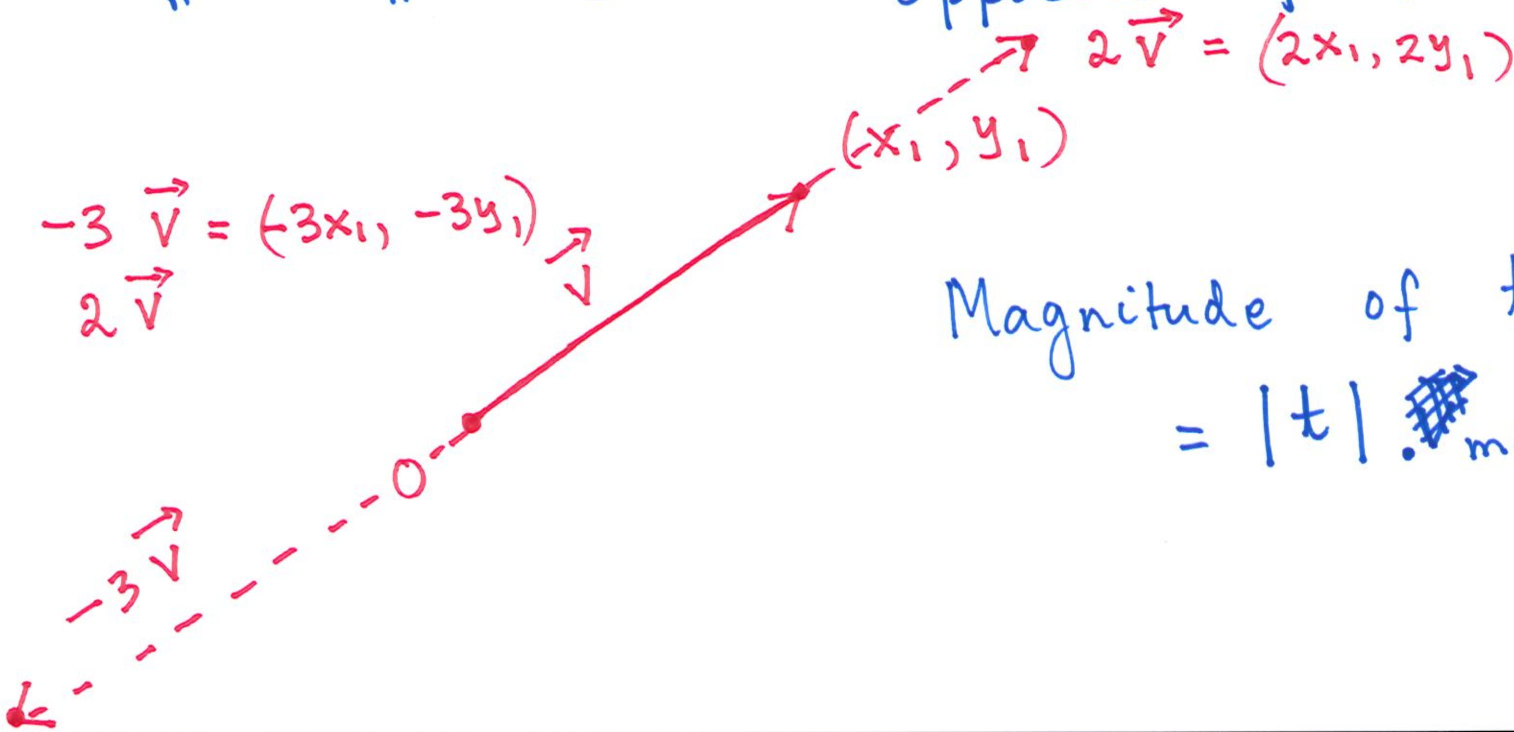
Scalar multiplication : Let $\underbrace{(x_1, y_1)}_{\vec{v}}$ be a vector

and let $0 \neq t \in \mathbb{R}$. Then $t\vec{v}$ is a vector.

(t → scalar)

Direction of $t\vec{v}$ is the same as $\vec{v}$ if $t > 0$

" " " " opposite of $\vec{v}$ if $t < 0$



Magnitude of $t\vec{v}$

$= |t| \cdot$ magnitude of $\vec{v}$

Properties of (vectors in x-y plane)

{x-y plane : $\mathbb{R}^2$, $\mathbb{R}^3$ - space x-y-z , $\mathbb{R}^n$ }

closed under addition & scalar multiplication

1. The sum of two vectors is another vector
2. Scalar multiple of a vector with a scalar is another vector
3. $\vec{V}_1 + \vec{V}_2 = \vec{V}_2 + \vec{V}_1$
4. If $a, b \in \mathbb{R}$ then

$$(a+b)\vec{V} = a\vec{V} + b\vec{V}$$

⋮

Besides "vectors" in a plane, many other mathematical objects satisfy similar properties.

For example:

space   vectors

the set of all matrices

the set of polynomials of degree n

→ addition of matrices

Later on (next class) we will see the formal definition of a vector space.