University of Idaho

MATH 430
Advanced Linear Algebra

Session 16

Review for Exam 1

(1) $V = \mathbb{R}^2$

$$(a_1, a_2) \oplus (b_1, b_2) = (2a_1 b_1, a_2 - b_2)$$

$$c \odot (a_1, a_2) = (a_1, ca_2)$$

Addition is not commutative : V is NOT a vector space

Zero vector = ? $\boxed{\text{No}}$.

Want to find $\boxed{(z_1, z_2)}$ s.t.

(i) $(a_1, a_2) \oplus (z_1, z_2) = (a_1, a_2)$ for any (a_1, a_2)

(ii) $(z_1, z_2) \oplus (a_1, a_2) = (a_1, a_2)$

(i) $\Rightarrow (2a_1 z_1, a_2 - z_2) = (a_1, a_2)$

(ii) $\Rightarrow (2z_1 a_1, z_2 - a_2) = (a_1, a_2)$

$$\left.\begin{aligned} a_2 - z_2 &= a_2 \\ z_2 - a_2 &= a_2 \end{aligned}\right\} \Rightarrow \nexists\ z_2 \text{ s.t. this holds for } \underline{\text{all}}\ (a_1, a_2)$$

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(6) $W = \{ (a_1, a_2, a_3) : a_1 + 2a_2 - 3a_3 = 1 \}$ $\subset \mathbb{R}^3$

$(0,0,0) \notin W$: $\underbrace{0 + 2\cdot 0 - 3\cdot 0}_{0} \neq 1$

$(0 \to a_1,\ 0 \to a_2,\ 0 \to a_3)$

W is NOT a subspace

$W = \{ a_1 + 2a_2 - 3a_3 = 0 \}$

(1) $0 + 2\cdot 0 - 3\cdot 0 = 0 \Rightarrow (0,0,0) \in W$

(2) $a \to (a_1, a_2, a_3)$ & $b \to (b_1, b_2, b_3) \in W$

✓ $a+b = (a_1+b_1,\ a_2+b_2,\ a_3+b_3) \in W$?

$(a_1+b_1) + 2(a_2+b_2) - 3(a_3+b_3)$

$= (a_1 + 2a_2 - 3a_3) + (b_1 + 2b_2 - 3b_3)$

$= 0 + 0$ because $a \in W$, $b \in W$

$= 0$

(3) ✓ $ca = (ca_1, ca_2, ca_3) \in W$?

$ca_1 + 2ca_2 - 3ca_3$

$= c\underbrace{(a_1 + 2a_2 - 3a_3)}_{=0} = c \cdot 0$ because $a \in W$

$= 0$.

(8) $S = \{ \overset{p_1}{x^3 + 2x^2 - x + 1}, \overset{p_2}{x^3 + 3x^2 - 1} \}$

$x^3 - 3x + 5 \in \underline{\text{span}}(S)$? Yes.

Find c_1, c_2 s.t.

$x^3 - 3x + 5 = c_1 p_1 + c_2 p_2$

Does c_1, c_2 exist ? Yes.

$c_1 = 3, \ c_2 = -2$

9 (b) $S = \{ \overset{p_1}{x^3 + 2x^2}, \overset{p_2}{-x^2 + 3x + 1}, \overset{p_3}{x^3 - x^2 + 2x - 1} \}$

Is S l.i. ? Yes.

Let $\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 = 0$. Solve for $\alpha_1, \alpha_2, \alpha_3$.

Is the zero solution $(\alpha_1 = \alpha_2 = \alpha_3 = 0)$ unique ? Yes

Can S span P_3 ? No.

$\dim(P_3) = 4 \neq$ size of $S = 3$

Field : $\mathbb{R}$

Any vector ~~vec~~ space V of dimension n is isomorphic to $\mathbb{R}^n$.

Let $\alpha = \{v_1, v_2, \ldots, v_n\}$ be a basis of V. Take $x \in V$.

$$x = \underline{\alpha_1} v_1 + \underline{\alpha_2} v_2 + \cdots + \underline{\alpha_n} v_n$$

$$\varphi : V \longrightarrow \mathbb{R}^n$$
$$x \longmapsto (\alpha_1, \alpha_2, \ldots, \alpha_n)$$

φ is an isomorphism.

$$x \sim (\alpha_1, \ldots, \alpha_n)$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\mathrm{Tr} = a+d = 0$$

$$a = -d$$

(19) Trace(A) = sum of the diagonal entries.

$2 = \overset{0}{\text{nullity}} + \text{rank}$

$N(T) \subseteq \mathbb{R}^2$ (domain)

Range $\subseteq \mathbb{R}^3$ (codomain)

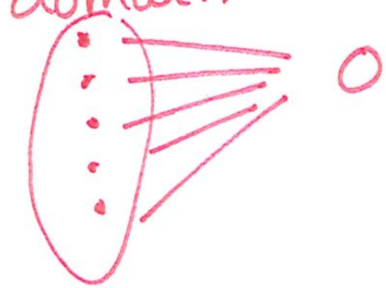
13 (a) $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$

$(a_1, a_2) \longmapsto (a_1 + a_2,\ 0,\ 2a_1 - a_2)$

$N(T) = \left\{ (a_1, a_2) : \boxed{\begin{array}{l} a_1 + a_2 = 0 \\ 2a_1 - a_2 = 0 \end{array}} \right\}$

Solve for a_1, a_2

domain



$a_1 = a_2 = 0$ $\qquad N(T) = \{ \vec{0} \}$

Dimension Thm: T is not-onto

$\Rightarrow$ T is one-to-one.

$R(T) \underset{\text{span}}{=} \left\{ T\begin{pmatrix} 1 \\ 0 \end{pmatrix}, T\begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} \underset{\text{span}}{=} \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\}$

$\dim(R(T)) = 2 < 3 = \dim(\mathbb{R}^3) \Rightarrow$ T is NOT onto.