Homework 1

MATH 430

All work must be shown clearly for full credit. You must justify all your answers.

Points will be deducted for incomplete/incorrect/haphazard/unorganized work.

Section 1.2

1. Let $V = \mathbb{R}^2$. Define addition and scalar multiplication as follows:

$$(a_1, a_2) \oplus (b_1, b_2) = (a_1 + b_1, a_2 b_2)$$
$$c \odot (a_1, a_2) = (ca_1, a_2)$$

Is V a vector space over $\mathbb{R}$? Justify your answer.

2. Let $V = \mathbb{R}^2$. For (a_1, a_2), $(b_1, b_2) \in V$ and $c \in \mathbb{R}$, define

$$(a_1, a_2) \oplus (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2)$$

and

$$c \odot (a_1, a_2) = (ca_1, ca_2).$$

Is V a vector space over $\mathbb{R}$? Justify your answer.

Section 1.3

3. Determine whether the following are subspaces of $\mathbb{R}^3$ under the operations of componentwise addition and scalar multiplication.

   (a) $W_1 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 = 3a_2 \text{ and } a_3 = -a_2)\}$
   (b) $W_2 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 = a_3 + 2\}$

4. Is the set $W = \{f(x) \in P(\mathbb{F}) : f(x) = 0 \text{ or } f(x) \text{ has degree } n)\}$ a subspace of $P(\mathbb{F})$ if $n \geq 1$? Justify your answer.