Homework 2

MATH 430

All work must be shown clearly for full credit. You must justify all your answers.
Points will be deducted for incomplete/incorrect/haphazard/unorganized work.

Section 1.4

1. Determine whether $x^3 - 3x + 5$ can be written as a linear combination of the polynomials in $\{x^3 + 2x^2 - x + 1, x^3 + 3x^2 - 1\}$.

2. Show that the matrices

$$\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \right\}$$

generate $M_{2\times 2}(\mathbb{F})$.

Section 1.5

3. Determine whether the following sets are linearly dependent or independent.

(a)

$$\{(1,1,0), (2,0,1), (0,1,3)\} \in \mathbb{R}^3$$

(b)

$$\left\{ \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ -4 & -4 \end{pmatrix} \right\} \in M_{2\times 2}(\mathbb{R})$$

4. Let M be a square upper triangular matrix with non-zero diagonal entries. Prove that the columns of M are linearly independent.

Section 1.6

5. The vectors $u_1 = (1,1,1,1)$, $u_2 = (0,1,1,1)$, $u_3 = (0,0,1,1)$ and $u_4 = (0,0,0,1)$ form a basis for $\mathbb{R}^4$. Find the unique representation of an arbitrary vector (a_1, a_2, a_3, a_4) in $\mathbb{R}^4$ as a linear combination of u_1, u_2, u_3, and u_4.